Can More Information Facilitate Communication?

Junichiro Ishida Takashi Shimizu

Osaka University Kansai University

June 2011

Results of This Paper

- We analyses a cheap talk model with a partially informed receiver
- We show that the more informed the receiver is, **the more efficient** information transmission via cheap talk can be possible
- This result is contrast to negative results in the previous literature

Intuition of Negative Results of the Previous Literature

- R's sensitivity to S's message is important to thwarts S's misreport
- The receiver has more information about the true state
  $\Rightarrow$ R becomes less responsive to sender's messages
  $\Rightarrow$ S has more incentive to misreport
- Informed receiver: Lai (2010), Ishida and Shimizu (2010), Moreno de Barreda (2010)
- Multiple partially informed senders: Austen-Smith (1993), Morgan and Stocken (2008), Galeotti, Chiglino, and Squintani (2009)

Intuition of Positive Result of This Paper

- S's messages play the dual role:
  - to provide information about the true state
  - to provide information about the reliability of the R's private information
- The 2nd role makes R more responsive when S sends a false message than when S sends a true message.
  $\Rightarrow$ S has less incentive to misreport

Players and Timing

- Baseline model: Crawford-Sobel uniform-quadratic model
- Sender(he) and Receiver(she)
- Nature chooses a state $\mathbf{t} \in [0, 1]$ according to the uniform distribution
- Each player receives his/her private signal
  S's signal is the realized state $\mathbf{t}$
- S sends a costless message $\mathbf{m} \in [0, 1]$ to R
- R chooses an action $\mathbf{a} \in \mathcal{R}$

Payoffs

- S's payoff: $\mathbf{U^S(t,a) = -(t + b - a)^2}$
- R's payoff: $\mathbf{U^R(t,a) = -(t - a)^2}$
- Bias: $\mathbf{b \in (0, 0.5)}$

Receiver's Signal

- R's signal: $\mathbf{r} \in [0, 1]$
- $\mathbf{P}(\mathbf{r} \in \mathbf{A}|\mathbf{t}) = \mathbf{q}\mathbb{I}(\mathbf{t} \in \mathbf{A}) + (1 - \mathbf{q})\lambda(\mathbf{A})$
  where λ: Lebesgue measure
- $\mathbf{q} \in [0, 1)$: accuracy of R's signal
- $\mathbf{q} = 0$: Crawford-Sobel case
- R cannot tell whether her signal is the true state or just a noise

Monotone Partition Equilibrium

- We use perfect Bayesian equilibrium as equilibrium concept
- Non-monotone equilibrium may exist (c.f. Chen 2009)
- We focus on monotone partition equilibria (MPE):
  - the state space $[0, 1]$ is partitioned by intervals
  - S informs R of which interval the true state is belonging to

Formal Definition of MPE

- A monotone partition strategy (MPS) is the sender's strategy $\{\mu_t\}_{t\in[0,1]}$ where there exists a partition of $[0,1]$, $\{T_i\}_{i\in I}$ such that
  - T_i is non-empty interval for any $i \in I$
  - $\mu_t = \mu_{t'}$ for any $i \in I$ and any $t, t' \in T_i$
  - $\text{Supp}\mu_t \cap \text{Supp}\mu_{t'} = \emptyset$ for any distinctive i, i' and any $t \in T_i$, $t' \in T_{i'}$
- A MPE is a PBE with MPS

Threshold Vector

- Proposition 1: Any MPE has at most finite intervals
- MPS (MPE) can be identified with a threshold vector: $\mathbf{t} = (\mathbf{t_1}, \dots, \mathbf{t_N})$ where $\mathbf{0} = \mathbf{t_0} < \mathbf{t_1} < \cdots < \mathbf{t_N} = \mathbf{1}$
- $\tau_{\mathbf{n}} := \mathbf{t_n} - \mathbf{t_{n-1}}$: length of **n**th interval

Receiver's Best Response

- $\mathbf{m_n}$: a message sent from $\mathbf{n}$th interval
- $\alpha(\mathbf{m_n}, \mathbf{r})$: R's pure strategy

$$\alpha(\mathbf{m_n}, \mathbf{r}) = \begin{cases} \frac{\mathbf{q}}{\mathbf{q}+(1-\mathbf{q})\tau_n}\mathbf{r} + \frac{(1-\mathbf{q})\tau_n}{\mathbf{q}+(1-\mathbf{q})\tau_n}\frac{\mathbf{t_n}+\mathbf{t_{n-1}}}{2} & \text{if } \mathbf{r} \in \mathbf{T_n} \\ \frac{\mathbf{t_n}+\mathbf{t_{n-1}}}{2} & \text{if } \mathbf{r} \notin \mathbf{T_n} \end{cases}$$

- When R's signal conflicts with S's message, R interprets her signal as a noise and ignore it

Equilibrium Condition

- $\boldsymbol{\Delta}(\mathbf{t}; \mathbf{t}_{n-1}, \mathbf{t}_n, \mathbf{t}_{n+1}) :=$
  $\mathbb{E}_t[\mathbf{U}^S(\mathbf{t}, \alpha(\mathbf{m}_{n+1}, \mathbf{r}))] - \mathbb{E}_t[\mathbf{U}^S(\mathbf{t}, \alpha(\mathbf{m}_n, \mathbf{r}))]$
- Necessary condition: $\forall \mathbf{n}$,

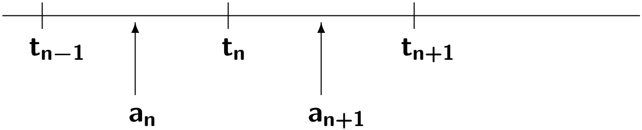
$$\boldsymbol{\Delta}(\mathbf{t}; \mathbf{t}_{n-1}, \mathbf{t}_n, \mathbf{t}_{n+1}) \begin{cases} \leq \mathbf{0} & \text{for } \mathbf{t} \in (\mathbf{t}_{n-1}, \mathbf{t}_n) \\ \geq \mathbf{0} & \text{for } \mathbf{t} \in (\mathbf{t}_n, \mathbf{t}_{n+1}) \end{cases}$$

- Sufficient condition: $\forall \mathbf{n}$,

$$\boldsymbol{\Delta}(\mathbf{t}; \mathbf{t}_{n-1}, \mathbf{t}_n, \mathbf{t}_{n+1}) \begin{cases} \leq \mathbf{0} & \text{for } \mathbf{t} < \mathbf{t}_n \\ \geq \mathbf{0} & \text{for } \mathbf{t} > \mathbf{t}_n \end{cases}$$

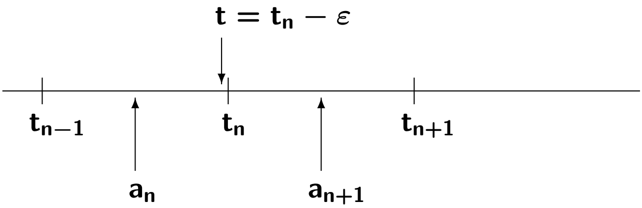
Intuition: Sender's Incentive in the Case of $\mathbf{q = 0}$

If the lengths of 2 neighboring intervals are the same,...



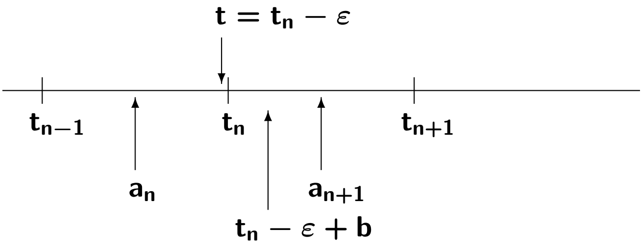
Intuition: Sender's Incentive in the Case of $\mathbf{q = 0}$

If the lengths of 2 neighboring intervals are the same,...



Intuition: Sender's Incentive in the Case of $\mathbf{q = 0}$

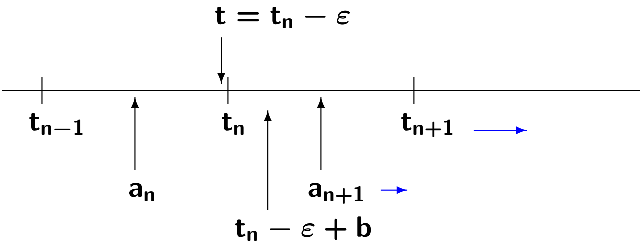
If the lengths of 2 neighboring intervals are the same,...



S has an incentive to tell a lie by sending $\mathbf{m_{n+1}}$

Intuition: Sender's Incentive in the Case of $\mathbf{q = 0}$

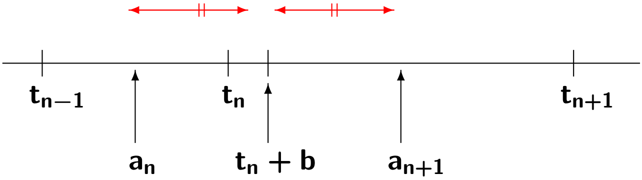
If the lengths of 2 neighboring intervals are the same,...



S has an incentive to tell a lie by sending $\mathbf{m_{n+1}}$

$\Rightarrow$ $\mathbf{t_{n+1}}$ must be higher

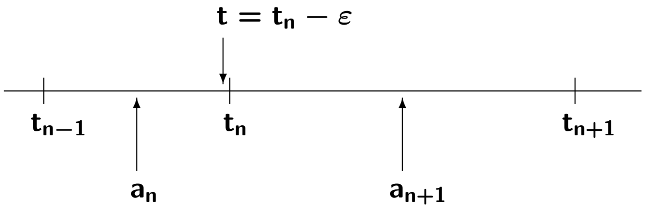
Intuition: Sender's Incentive in the Case of $\mathbf{q = 0}$



Equilibrium condition: $\tau_{n+1} = \tau_n + 4b$

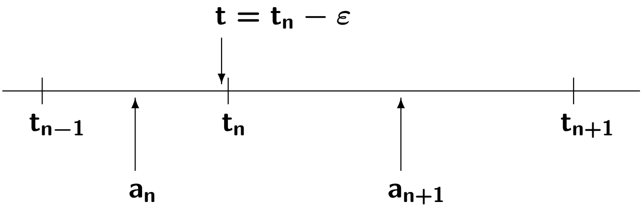
Intuition: Sender's Incentive in the Case of $\mathbf{q > 0}$

Intuition: Sender's Incentive in the Case of $\mathbf{q} > \mathbf{0}$



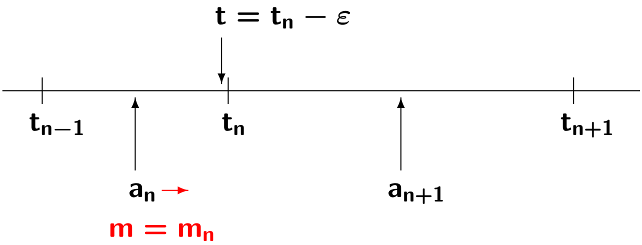
Intuition: Sender's Incentive in the Case of $\mathbf{q > 0}$

On the event $\mathbf{r = t}$,...



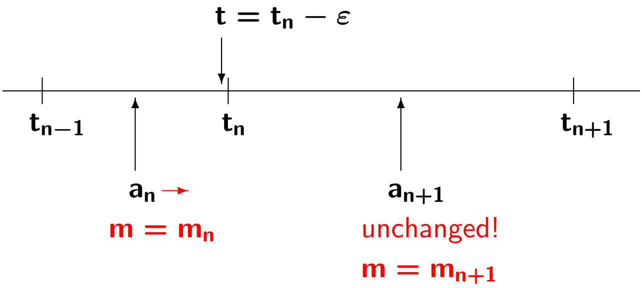
Intuition: Sender's Incentive in the Case of $\mathbf{q} > \mathbf{0}$

On the event $\mathbf{r} = \mathbf{t}$,...



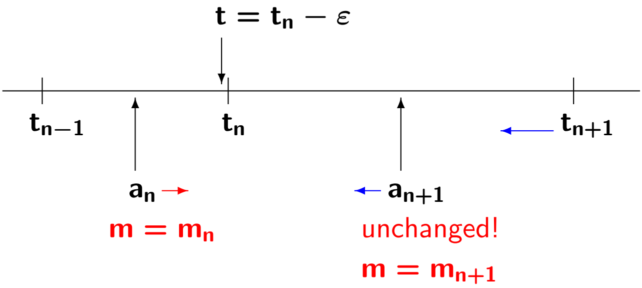
Intuition: Sender's Incentive in the Case of $\mathbf{q > 0}$

On the event $\mathbf{r = t}$,...



Intuition: Sender's Incentive in the Case of $\mathbf{q} > 0$

On the event $\mathbf{r} = \mathbf{t}$,...



$\mathbf{t_{n+1}}$ can be lower!

Intuition: Sender's Incentive in Moreno de Barreda (2010)

$\mathbf{r \backsim U[t - c, t + c]}$

Intuition: Sender's Incentive in Moreno de Barreda (2010)

$\mathbf{r} \backsim \mathbf{U[t-c, t+c]}$

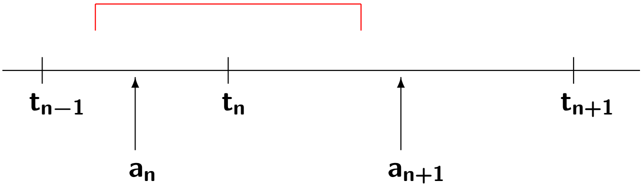
When $\mathbf{c} < \frac{1}{2}\tau_{\mathbf{n+1}}, \ldots$

Intuition: Sender's Incentive in Moreno de Barreda (2010)

$\mathbf{r \backsim U[t - c, t + c]}$

When $\mathbf{c} < \frac{1}{2}\tau_{\mathbf{n+1}}, \ldots$

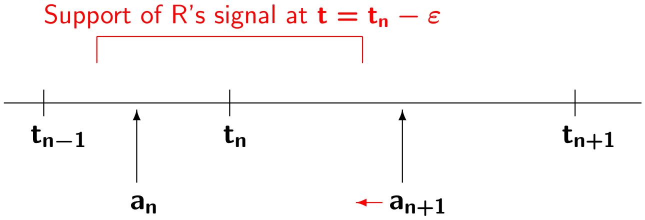
Support of R's signal at $\mathbf{t = t_n} - \varepsilon$



Intuition: Sender's Incentive in Moreno de Barreda (2010)

$\mathbf{r \backsim U[t - c, t + c]}$

When $\mathbf{c} < \frac{1}{2}\tau_{\mathbf{n+1}}, \ldots$



S has an incentive to lie!

Proposition 2: Equilibrium Condition

- $\mathfrak{t}$ is MPE iff $G^+(\tau_n, \tau_{n+1}) \geq 0$ and $G^-(\tau_n, \tau_{n+1}) \leq 0$ for any n where

$$
\begin{aligned}
G^+(\tau_n, \tau_{n+1}) := {} & (\tau_{n+1} + \tau_n)(\tau_{n+1} - \tau_n - 4b) \\
& + \frac{q^2(1-q)\tau_{n+1}^3}{3\left(q + (1-q)\tau_{n+1}\right)^2} + \frac{2q^2(1-q)\tau_n^3}{3\left(q + (1-q)\tau_n\right)^2} \\
& + \frac{q^2\tau_n(\tau_n + 4b)}{q + (1-q)\tau_n} \\
G^-(\tau_n, \tau_{n+1}) := {} & (\tau_{n+1} + \tau_n)(\tau_{n+1} - \tau_n - 4b) \\
& - \frac{2q^2(1-q)\tau_{n+1}^3}{3\left(q + (1-q)\tau_{n+1}\right)^2} - \frac{q^2(1-q)\tau_n^3}{3\left(q + (1-q)\tau_n\right)^2} \\
& - \frac{q^2\tau_{n+1}(\tau_{n+1} - 4b)}{q + (1-q)\tau_{n+1}}
\end{aligned}
$$

Proposition 2 Equilibrium Condition (cont'd)

- Let $\underline{\tau}(\tau_n; q)$ be the unique solution $\tau_{n+1} \in [0, \infty)$ of $G^+(\tau_n, \tau_{n+1}) = 0$ and $\overline{\tau}(\tau_n; q)$ be the unique solution $\tau_{n+1} \in [2b, \infty)$ of $G^-(\tau_n, \tau_{n+1}) = 0$. Then $\mathfrak{t}$ is MPE iff $\underline{\tau}(\tau_n, q) \leq \tau_{n+1} \leq \overline{\tau}(\tau_n, q)$ for any n
- $\underline{\tau}(\tau_n; 0) = \overline{\tau}(\tau_n; 0) = \tau_n + 4b$
- $\underline{\tau}(\tau_n; q) < \tau_n + 4b < \overline{\tau}(\tau_n; q)$ for $q > 0$

Direct and Indirect Effects of Increase of $\mathbf{q}$

- $\mathbf{V^i(\mathfrak{t};q)}$: Player $\mathbf{i}$'s expected payoff given $\mathfrak{t}$
- Proposition 3: $\partial \mathbf{V^i}/\partial \mathbf{q} > \mathbf{0}$ for $\mathbf{i = S, R}$
- Definition 2(i): $\mathfrak{t}$ *facilitates communication* rather than $\mathfrak{t}'$ under $\mathbf{q}$ if $\mathbf{V^i(\mathfrak{t};q) > V^i(\mathfrak{t}';q)}$ for $\mathbf{i = S, R}$
- Definition 2(ii): $\mathbf{q}$ *facilitates communication* rather than $\mathbf{q'}$ if for any MPE $\mathfrak{t}'$ under $\mathbf{q'}$, there exists an MPE $\mathfrak{t}$ under $\mathbf{q}$ such that $\mathfrak{t}$ facilitates communication rather than $\mathfrak{t}'$ under $\mathbf{q'}$

With or Without Receiver's Private Information

- Proposition 4:
  - For any MPE $\mathfrak{t}$ under $\mathbf{q} = \mathbf{0}$, it is also an MPE under $\mathbf{q} > \mathbf{0}$
  - Furthermore, any $\mathbf{q} > \mathbf{0}$ facilitates communication rather than $\mathbf{q} = \mathbf{0}$
- Remark 1: Under $\mathbf{q} > \mathbf{0}$ there may exist an MPE with more intervals than those under $\mathbf{q} = \mathbf{0}$

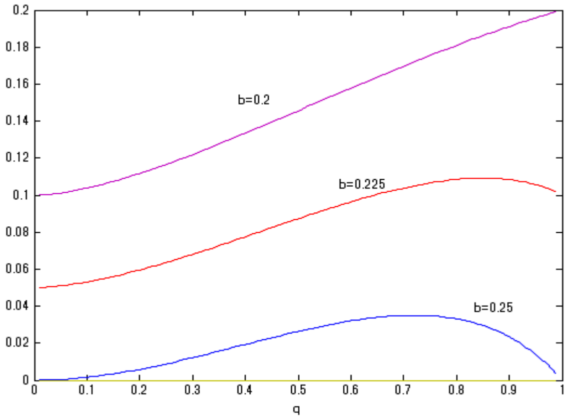
Example 1 (Difficulty of Characterization of MPE)

- When $\mathbf{b}$ is smaller than but close to $\mathbf{1/8}$, there exists an MPE with 2 intervals and $\tau_1 \geq \tau_2$ for sufficiently large $\mathbf{q}$
- The condition $\tau_{n+1} = \underline{\tau}(\tau_n; \mathbf{q})$ may be violated in the most efficient MPE

More Information Facilitates Communication (at least, in the case of small q)

- Lemma 7: Under sufficiently small q, $\underline{\tau}(\tau; q) > \tau$
  $\Rightarrow$ In the most efficient MPE, $\tau_{n+1} = \underline{\tau}(\tau_n; q)$ for any n
- Lemma 8: Under sufficiently small q, $\underline{\tau}$ is strictly decreasing in q
- Proposition 5: There exists $\overline{q} > 0$ s.t. any $q \in (0, \overline{q}]$ facilitates communication rather than any $q' < q$

The length of the 1st interval in the most efficient MPE with 2 intervals



What We Do Not Know Yet

- What is happening in the case of large $\mathbf{q}$
- Sufficient condition for excluding non-monotone equilibria
- What will happen in more general class of information structures