Cheap Talk with an Informed Receiver

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Abstract

This paper examines the effectiveness of cheap talk when the receiver is imperfectly informed. We show that the receiver's prior knowledge becomes an impediment to efficient communication: the more the receiver is informed, the less information she can extract from the sender. In fact, when the receiver is as informed as the sender, perfect revelation of private information via cheap talk is not feasible for an arbitrarily small degree of preference incongruence. This draws sharp contrast to the conventional counterpart which always yields a fully separating equilibrium as long as the preferences are sufficiently congruent. We also discuss some implications for organizational design.

Keywords: Cheap talk, Informed receiver, Truth telling, Rational ignorance.

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1 Introduction

Since the seminal work of Crawford and Sobel [7], a substantial amount of attention has been paid to strategic aspects of communication between players with conflicting interests. A canonical model of this literature, which is now widely known as "cheap talk," typically has an informed sender who communicates with an uninformed receiver by sending a costless message.[1] In this setting, the amount of information which can be conveyed to the receiver depends on the degree of preference congruence between the sender and the receiver. In general, less information can be conveyed when the two players' preferences diverge away from each other: when the receiver does not trust the sender's ulterior motive, any message cannot be taken at face value, and communication necessarily becomes less informative. This original insight has been fruitfully applied to many distinct issues, not just in economics but also in other disciplines such as political science, and has now become an indispensable tool to analyze various settings which involve communication among concerned parties.[2]

Despite several notable developments that have been made over the years, however, most existing works focus on the case where the sender who knows the state of nature with precision sends a message to the receiver who knows nothing about it. The latter assumption, that the receiver has no information of her own, seems particularly restrictive as the receiver often has her own sources of information. In this paper, we show that the receiver's ability to gain some information on her own, aside from the conflict of interests, can be a major impediment to effective communication. The situation we consider is as follows. The sender observes a signal which imperfectly reflects the true state and then sends a costless message to the receiver. Upon receiving the message, the receiver then "double-checks" it by using her own source of information and chooses some action. Under this setup, we show that communication cannot be informative for an arbitrarily small preference incongruence when the receiver's source of information is as reliable as the sender's. In other words, no information can be

[1] Recently, several attempts are made to explicitly incorporate the cost of lying into the setting of cheap talk. Most notably, see Kartik et al. [14] and Kartik [13].

[2] Some notable applications of cheap talk include Farrell and Gibbons [10] and Matthews and Postlewaite [17] for bargaining (double-auction) problems, Matthews [18] and Austen-Smith [1] for political debates, and Dessein [9] for organizational design.

conveyed via cheap talk between two parties with almost identical preferences when they are equally informed. This draws sharp contrast to the conventional setup where there is always an informative equilibrium as long as the preference incongruence is sufficiently small.

There is a very simple and intuitive reason for this result. The potential cost of misreporting is that, by doing so, it generally results in a less efficient choice of action, which lowers the welfare of both players. This is more of a concern when the receiver has absolutely no idea and hence must rely on the sender's recommendation rather blindly: since the clueless receiver follows whatever the sender insists, he has an incentive to report truthfully as long as the preferences are sufficiently congruent. The situation changes drastically, however, when the receiver has her own sources of information because the receiver can now overrule the sender's recommendation based on her own information. The problem here is that this overruling does not occur randomly: the receiver overrules the sender's recommendation when she believes, judging from her own information, that it is more likely to be wrong. The receiver thus effectively functions as a gatekeeper to sort out bad information, but this capability, or the lack of commitment not to use her own information, clearly diminishes the sender's incentive to report truthfully. Since the sender knows that he would be corrected whenever he is way off the mark, the salience of the private benefit is magnified, which renders communication less informative. This result means that the overall efficiency may improve when the receiver is totally inept at collecting information on her own and dependent exclusively on the sender's recommendation.

The present setup has some inherent connection to cheap-talk models when there are multiple informed players. Among them, the paper is most closely related to Morgan and Stocken [19], Galeotti et al. [11], Lai [16] and Moreno de Barreda [8]. The first two consider models with multiple imperfectly informed senders,[3] while the latter two consider cases with an imperfectly informed receiver.[4] Although each of these has its own focus and does not

[3] Austen-Smith [2] considers a case with multiple imperfectly informed experts with a special focus on the optimal forum for giving advice, i.e., whether information should be disclosed simultaneously or sequentially. There are also works which consider multiple perfectly informed senders, e.g., Gilligan and Krehbiel [12], Krishna and Morgan [15], and Battaglini [3].

[4] Some early attempts to model an informed receiver are provided by Seidmann [20] and Watson [21]. Chen [5] and Chen [6] for more recent works on models with an imperfectly informed receiver. In a different vein, Blume et al. [4] introduce communication errors and show that adding noise can improve welfare.

necessarily focus on the question we pose here, these studies theoretically have one common feature: the incentive for truthful communication diminishes as the receiver, broadly defined, accumulates more information.[5] This is because when the receiver has more information of her own, she becomes less sensitive to the sender's message. In order to sway the receiver towards the sender's preferred position, therefore, the sender must exaggerate more, rendering communication less informative. Alternatively, the receiver's insensitivity to information reduces the effective range of the action space, which yields an equivalent effect to an increase in the preference bias. This means that the original insight of Crawford and Sobel [7] directly applies to each of these cases: in fact, in those models, truthful communication is always possible as long as the preferences are sufficiently congruent.[6]

The fundamental logic of our model is a little different, as we detailed above. The driving force of our model is the informed receiver's asymmetric and selective response to the sender's message, depending on its perceived quality, which substantially reduces the cost of lying. With this difference, we show that there is a case where no information can be conveyed via cheap talk for an arbitrarily small degree of preference incongruence. The most distinctive feature of our model is the role played by the information gap between the sender and the receiver: in our setting, the nature of equilibrium changes drastically when the receiver's prior information becomes as accurate as the sender's, whereas there are no such tipping points in the previous models where the information gap carries no special meanings.[7] This aspect of our model suggests a key departure from the previous literature which has identified the conflict of interests almost exclusively as the impediment to effective communication.

The paper proceeds as follows. Section 2 provides a motivating example with two signals, hopefully to clarify what we try to get at in this paper. Section 3 extends the analysis to

[5] Chen [5] also obtains a similar result in a model with an imperfectly informed receiver although that is not her main focus.

[6] To be more precise, in Morgan and Stocken [19] and Galeotti et al. [11], a player becomes more informed when he communicates with more players. They then show that there is an upperbound of the number of players to communicate with. In either case, this upperbound goes to infinity when the preference bias goes to zero.

[7] To be precise, it should be noted that it is not our main focus to emphasize that communication cannot be made informative when the accuracy of the receiver's information gets past that of the sender's, which is more of an artifact of our model specification. Our main point is rather that the difference in the accuracy of information matters for the effectiveness of communication.

incorporate an arbitrary finite number of signals and shows that the basic insight obtained in section 2 holds in a more general environment. Section 4 discusses some implications for organizational design by slightly extending the model. Finally section 5 makes some concluding remarks.

2 An illustrative example

2.1 The setup

We consider a simple model of cheap talk to illustrate how the receiver's imperfect knowledge becomes an impediment to effective communication. There are two players, Player 0 (male) and Player 1 (female), to whom we interchangeably refer as the sender and the receiver, respectively, throughout the analysis. The game proceeds as follows:

1. Nature randomly chooses the state $t \in \{0, 1\}$ with equal probability, i.e.,

$$P(t = 0) = P(t = 1) = 0.5.$$

2. Player n receives a private signal $s_n \in \{0, 1\}$. We assume that s_0 and s_1 are drawn independently and

$$P(s_n = k|t = k) = r_n \in [0.5, 1] \quad n, k = 0, 1.$$

3. Upon observing s_0, Player 0 costlessly sends a message $m \in \{0, 1\}$ to Player 1.

4. Upon observing s_1 and m, Player 1 chooses an action $a \in \{0, 1\}$.

5. Player n receives a payoff $u_n(t, a)$ such that

$$u_n(t, a) = \mathbb{I}(a = t) + b_n\mathbb{I}(a = n),$$

where $\mathbb{I}$ is the indicator function, and b_n is Player n's private benefit of implementing his or her preferred action.

The accuracy of Player n's prior information is given by r_n where $r_n = 0.5$ means that Player n's signal contains no information. Since the case with a totally uninformed sender is irrelevant, we assume throughout the analysis that $r_0 > 0.5$. In this specification, $r_0 - r_1$ represents what we call the information gap.

Each player has his or her own preferred action, possibly arising from some private benefits associated with each action, so that their preferences are not perfectly aligned. The degree of preference incongruence is measured by b_n, which we refer to as the preference bias for the ease of exposition.[8] Let $\beta_n := (1 - b_n)/2$, where we assume $b_n \in (0, 1)$ or, equivalently, $\beta_n \in (0, 0.5)$. For this example, we suppose that the preference biases are symmetric so that $b_0 = b_1 = b$ and $\beta_0 = \beta_1 = \beta$.

2.2 The condition for truth telling

For most of the analysis, we restrict our attention to pure-strategy equilibria, especially fully separating equilibria in which the sender reports his observation truthfully. We start with the problem faced by the receiver who observes her own signal s_1 and also receives a message m from the sender. Define $P_{ij} := P(t = 1|s_0 = i, s_1 = j)$, which coincides with the receiver's posterior belief when the sender reveals truthfully. Let $A(m, s_1)$ denote the receiver's strategy for a given set of available information (m, s_1). Once the posterior belief is obtained, the receiver's choice of action is characterized by the following simple rule:

$$A(i, j) = \begin{cases} 1 & \text{if } P_{ij} > \beta, \\ 0 & \text{if } P_{ij} < \beta. \end{cases} \tag{1}$$

Taking the receiver's strategy A as given, it is now straightforward to obtain the condition for truth telling. A necessary, and naturally more stringent, condition for truth telling is that the sender must have an incentive to disclose his observation truthfully when he observes $s_0 = 1$, i.e.,

$$\mathbb{E}[u_0(t, A(1, s_1))|s_0 = 1] \geq \mathbb{E}[u_0(t, A(0, s_1))|s_0 = 1].$$

[8] Note the difference from the standard setting, such as Crawford and Sobel [7], where the degree of preference incongruence is measured by the distance between the bliss points.

This condition can be written as

$$r_0\Bigg(r_1\Big(u_0(1,A(1,1))-u_0(1,A(0,1))\Big)+(1-r_1)\Big(u_0(1,A(1,0))-u_0(1,A(0,0))\Big)\Bigg)\geq$$
$$(1-r_0)\Bigg(r_1\Big(u_0(0,A(0,0))-u_0(0,A(1,0))\Big)+(1-r_1)\Big(u_0(0,A(0,1))-u_0(0,A(1,1))\Big)\Bigg). \tag{2}$$

There exists a fully separating (truth-telling) equilibrium when this condition is satisfied.

2.3 The role of the information gap in inducing truth telling

The cheap-talk literature has identified preference incongruence almost exclusively as the impediment to effective communication. In general, communication becomes more informative as the preferences are more closely aligned. In a discrete setup such as this one, it is expected that there is a threshold level of the preference bias b below which perfect revelation of private information via cheap talk is feasible. We first confirm that this conjecture holds true when the receiver does not have her own source of information in addition to the sender's message, as is normally the case in most cheap-talk models.

Formally, the receiver is totally uninformed when $r_1 = 0.5$. Given that $r_0 > 0.5$, we have $A(m, s_1) = m$ if $2r_0 - 1 > b$. The condition for truth telling (2) then becomes

$$r_0(1-b) \geq (1-r_0)(1+b) \;\Leftrightarrow\; 2r_0 - 1 \geq b,$$

meaning that perfect revelation of private information is feasible for any $r_0 > 0.5$ as long as the preference bias is sufficiently small. In other words, communication can be made fully informative when the preferences are sufficiently congruent. Apparently, there should be nothing surprising about this result since any message sent by the sender can be taken at face value when the conflict of interests is less severe.

The underlying logic we have seen just above seems so robust that it holds for any given information structure (r_0, r_1) even when $r_1 > 0.5$. As it turns out, though, this is not always the case in the current setup. We now let the receiver be endowed with her own source of information and see how the receiver's imperfect prior knowledge obstructs information transmission. We are especially interested in the limiting case where $r_1 \to r_0$, i.e., the information gap converges to zero. In this case, note that $\lim_{r_1 \to r_0} P_{01} = \lim_{r_1 \to r_0} P_{10} = 0.5$, which implies that $A(1,1) = A(0,1) = A(1,0) = 1$ for any given positive bias b. For communication to have any value, therefore, one must have $A(0,0) = 0$, which is the case if b is sufficiently small. Given this, (2) can be written as

$$r_0(1-r_1)(1-b) \geq (1-r_0)r_1(1+b) \;\Leftrightarrow\; \frac{r_0 - r_1}{(1-r_0)r_1 + r_0(1-r_1)} \geq b. \tag{3}$$

An increase in r_1 towards r_0 lowers the maximum bias below which truthful communication is feasible. This suggests that the receiver faces a cumbersome tradeoff between the quality of her own information and the quality of communication: an increase in r_1 by itself contributes to better decision making but may deteriorate the quality of information that can be extracted from the sender. When $r_1 = r_0$, this condition cannot be satisfied by any positive b, no matter how small it is.

To understand this result, one must look at, as always, the cost and benefit of misrepresenting information. First, the benefit is relatively clear, as the sender may sway the receiver towards his preferred action by making a biased recommendation. This benefit is represented by $(r_0(1-r_1) + (1-r_0)r_1)b$, which is strictly positive for any $b > 0$ and (r_0, r_1). This information manipulation comes at a cost, however, because it necessarily entails an inefficient use of information. Given the receiver's strategy, a potential loss arises when the receiver observes $s_1 = 0$, in which case the sender's message becomes pivotal. That is, the cost of misrepresenting information is determined by how much the sender loses by recommending his preferred action when $s_0 = 1$ and $s_1 = 0$, which is precisely captured by P_{10}. As the receiver's information becomes as accurate as the sender's, P_{10} converges towards one half. At this point, therefore, it is a fair bet, one way or the other, as far as the probability of choosing the right action is concerned. The sender can only gain from lying by the margin of the preference bias, thereby extinguishing any hope for informative communication.

2.4 Equilibrium with an informed receiver

The argument made thus far indicates how the receiver's prior knowledge alters the nature of communication in strategic environments. In particular, when the receiver is as informed as the sender, the condition for truth telling cannot be satisfied for an arbitrarily small preference bias b. More precisely, we can establish the following result.[9]

Proposition 1 Define $R \in [0.5, r_0)$ such that $\beta = P_{01}|_{r_1=R}$. If $r_0 \geq 1 - \beta$, then

1. perfect revelation of private information (full separation) is feasible for $r_1 \leq R$, and

2. no private information can be transmitted to the receiver for $r_1 > R$.

If $r_0 < 1 - \beta$, no private information can be transmitted to the receiver.

Proof: See Appendix.

The proposition implies some interesting properties. Before we see them, we first consider a benchmark case where the receiver is totally uninformed as normally assumed in prototype cheap-talk models. In this benchmark case, as can easily be expected, truth telling can be induced on the equilibrium path as long as the preference bias is sufficiently small.

Corollary 1 Suppose that $r_1 = 0.5$. Then, for any given $r_0 \in (0.5, 1]$, there exists a threshold $\bar{b}(r_0) := 2r_0 - 1$ such that perfect revelation of private information is feasible if and only if $\bar{b}(r_0) \geq b$.

We now let the receiver be endowed with some information of her own, so that $r_1 > 0.5$. As we have seen, the condition for truth telling in pure strategies cannot be satisfied when $r_0 = r_1$. In fact, one can even make a stronger statement in this particular example: with a positive preference bias, communication cannot be informative at all for any $r_1 \geq r_0$, no matter how slight the bias is.

Corollary 2 No private information can be transmitted to the receiver for any $b > 0$ if $r_1 \geq r_0$.

The result implies a cumbersome tradeoff for the receiver. On one hand, there is an apparent gain from being better informed: as her own signal becomes more accurate, she can

[9]The result holds even if the strategy space is extended to include mixed strategies. See Appendix for details.

make a better prediction when the quality of communication is held constant. On the other hand, there is also a downside because an improvement in the receiver's prior information may deteriorate the quality of communication. Due to this tradeoff, the receiver's expected payoff can be non-monotonic in r_1, i.e., there may exist a range where an increase in r_1 lowers the receiver's welfare.

Corollary 3 Suppose that $r_0 > 1-\beta$. Then, the receiver's ex ante largest equilibrium payoff $U_1(r_1)$ is as follows: if $R < 1-\beta$,

$$U_1(r_1) = \begin{cases} r_0 + \frac{b}{2} & \text{if } r_1 \leq R \\ \frac{1}{2} + b & \text{if } R < r_1 \leq 1-\beta \\ r_1 + \frac{b}{2} & \text{if } r_1 > 1-\beta. \end{cases}$$

If $R \geq 1-\beta$,

$$U_1(r_1) = \begin{cases} r_0 + \frac{b}{2} & \text{if } r_1 \leq R \\ r_1 + \frac{b}{2} & \text{if } r_1 > R. \end{cases}$$

In either case, U_1 is not monotone in r_1.

Proof: Since the receiver simply maximizes her expected payoff given the message m_0 and the signal s_1, her *ex ante* payoff depends only on the amount of information conveyed from the sender. This means that if there exists a fully separating equilibrium, the receiver cannot improve her payoff any better. For other cases, we already know that there only exist uninformative equilibria. Computing the expected payoff in each type of equilibrium then yields the result. ∎

This simple example sheds light on an important aspect of communication: in a strategic situation with conflicting interests, it is increasingly difficult to exchange meaningful information when concerned parties are equally (and imperfectly) informed. In the next section, we extend this argument to a more general setting and obtain conditions under which communication ceases to work.

3 The general model

3.1 The setup

We consider basically the same model as in the previous section but enrich its information structure to incorporate arbitrary finite sets of private signals. Let $S_n = \{0, 1, \ldots, K_n\}$ denote the set of signals for Player n and ℓ_n denote the likelihood ratio function where

$$\ell_n(i) = \frac{P(s_n = i|t = 1)}{P(s_n = i|t = 0)}.$$

The message space $\mathcal{M}$ is also extended accordingly so that $\mathcal{M} = S_0$. Without loss of generality, we order the signals by the likelihood ratio:

$$\begin{cases} \ell_0(0) > \ell_0(1) > \cdots > \ell_0(K_0), \\ \ell_1(0) > \ell_1(1) > \cdots > \ell_1(K_1). \end{cases} \tag{4}$$

For the ease of exposition, we often refer to $(P(s_n|t))_{s_n \in S_n, t=0,1}$ as Player n's *signal structure*. In this setup, we also allow for the possibility that $b_0 \neq b_1$ and $P(t = 0) \neq P(t = 1)$.

We denote the sender's strategy by $M : S_0 \to \mathcal{M}$. Also, as above, let $A : \mathcal{M} \times S_1 \to \{0, 1\}$ denote the receiver's strategy. In this setup, we seek for an equilibrium in which communication matters, i.e., an equilibrium in which the sender reveals his private information truthfully and the receiver's choice of action depends on the sender's message with positive probability.[10] We in particular focus on what we call fully informative equilibria defined as follows.

Definition 1 An equilibrium is said to be *fully informative* if and only if

(i) $M(s_0) = s_0$ for any $s_0 \in S_0$,

(ii) there exist at least one $j \in S_1$ such that $A(i, j) \neq A(i', j)$, $i \neq i'$.

3.2 The equilibrium conditions

Since the nature of the problem virtually remains the same, the equilibrium conditions only need some slight modifications, mostly generalizations, from the two-signal case. Given the

[10]The latter requirement is necessary to exclude truth-telling equilibria in which the receiver's action is independent of the sender's message, and communication hence plays no role. If the receiver's action is independent of the sender's message, any strategy, including the truth-telling one, trivially constitutes an equilibrium.

sender's strategy, define

$$\mathcal{S}_0(m) = \{i \in S_0 | M(i) = m\},$$

as the set of the sender's types sending a message m. With this definition, we can obtain a generalized version of (1).

Lemma 1 Given the sender's strategy M, the receiver's best response is

$$A(m,j) = \begin{cases} 1 & \text{if } \frac{\sum_{i \in \mathcal{S}_0(m)} P_{ij} P(s_0=i|s_1=j)}{\sum_{i \in \mathcal{S}_0(m)} P(s_0=i|s_1=j)} > \beta_1, \\ 0 & \text{if } \frac{\sum_{i \in \mathcal{S}_0(m)} P_{ij} P(s_0=i|s_1=j)}{\sum_{i \in \mathcal{S}_0(m)} P(s_0=i|s_1=j)} < \beta_1. \end{cases} \tag{5}$$

Proof: See Appendix.

Similarly, we also investigate the sender's problem to obtain a generalized version of (2). To this end, define

$$\mathcal{S}_1(i,i') = \{j | A(M(i),j) = 1, A(M(i'),j) = 0\},$$

as the set of signals for which the receiver changes her choice of action in response to the sender's message on the equilibrium path. We then obtain the following result.

Lemma 2 If receiver's strategy A satisfies (5), then for any $i, i' \in S_1$, either one of the followings holds:

(i) $\mathcal{S}_1(i,i') = \mathcal{S}_1(i',i) = \emptyset$.

(ii) $\mathcal{S}_1(i,i') \neq \emptyset$ and $\mathcal{S}_1(i',i) = \emptyset$.

(iii) $\mathcal{S}_1(i,i') = \emptyset$ and $\mathcal{S}_1(i',i) \neq \emptyset$.

Moreover,

$$\mathbb{E}\left[u_0(t, A(M(i),j)) | s_0 = i\right] \geq \mathbb{E}\left[u_0(t, A(M(i'),j)) | s_0 = i\right]$$

holds if and only if

$$\begin{cases} \text{(i) holds,} \\ \text{(ii) and } \beta_0 \geq \dfrac{\sum_{j \in \mathcal{S}_1(i,i')} (1-P_{ij}) P(s_1 = j | s_0 = i)}{\sum_{j \in \mathcal{S}_1(i,i')} P(s_1 = j | s_0 = i)} \text{ hold, or} \\ \text{(iii) and } \beta_0 \leq \dfrac{\sum_{j \in \mathcal{S}_1(i',i)} (1-P_{ij}) P(s_1 = j | s_0 = i)}{\sum_{j \in \mathcal{S}_1(i',i)} P(s_1 = j | s_1 = i)} \text{ hold.} \end{cases} \tag{6}$$

Proof: See Appendix.

These results straightforwardly lead to the conditions that must be satisfied by any equilibrium in this generalized setting. A pair of strategies (M, A) constitutes an equilibrium if and only if

- (5) holds for any $m \in \mathcal{M}$ such that $\mathcal{S}_0(m) \neq \emptyset$ and any $j \in S_1$, and
- (6) holds for any $i, i' \in S_0$.

3.3 Informative equilibria with arbitrary finite sets of signals

In this setup, one may conjecture that perfect revelation of information is feasible as long as the preference biases b_n are sufficiently small. As we have seen in the example above, however, this conjecture is not generally true: no separating equilibria exist in the two-signal example when the signal structures satisfy a certain condition, namely r_1 being sufficiently close to r_0. We now extend this argument to this generalized setting and obtain more general conditions under which perfect revelation of private information via cheap talk cannot be attained.

As in the two-signal example, we continue to restrict our attention to fully informative equilibria, i.e., fully separating equilibria in which "communication matters." Under this definition, we can make the following statement which is our main result.

Condition 1 Each $j \in S_1$ satisfies, at least, one of the followings:

(i) there exists $i \in S_0$, denoted as $i(j)$, such that $1 - \beta_0 > P_{i(j).j} > \beta_1$,

(ii) $P_{ij} > \beta_1 \quad \forall i$, or

(iii) $P_{ij} < \beta_1 \quad \forall i$.

Theorem 1 No fully informative equilibria exist under Condition 1.

Proof: See Appendix.

When a signal j satisfies either (ii) or (iii) of Condition 1, the receiver's own signal is so accurate that there is no need to rely on the sender's information. When every signal $j \in S_1$ satisfies either (ii) or (iii), the receiver's choice of action is totally independent of the sender's message and any M, including the truth-telling one, is hence weakly optimal for the sender. This is apparently a trivial and uninteresting case since communication actually plays no role.

The situation becomes more intriguing, therefore, when there exists some $j \in S_1$ for which neither (ii) nor (iii) holds: this is the case where the sender's message can be influential and communication plays some role. Even in this case, however, perfect revelation of private information cannot be attained if that signal satisfies (i). This is a situation where the sender's message roughly cancels out the receiver's signal, so that the resulting posterior belief falls into the range where the conflict of interest between the sender and the receiver is at its pinnacle: when the posterior belief falls into this intermediate range, the players can never agree on which action to take even with complete information. Theorem 1 shows that when that happens, it is impossible to achieve full separation irrespective of the degree of preference incongruence.

To understand Theorem 1 more intuitively, it is instructive to consider the two-signal example presented in the previous section. As we have already seen, no fully informative equilibria exist in this example when $r_0 = r_1$. We now show that the signal structures in this example satisfy Condition 1, which can best be seen graphically by Figure 1.[11] In this example, $S_1 = \{0, 1\}$ and either (i), (ii) or (iii) of Condition 1 must be satisfied for each $s_1 \in S_1$: first, $s_1 = 0$ satisfies (i) because there is a signal $s_0 = 1$ with the corresponding posterior belief P_{12} that falls into the intermediate range; second, $s_1 = 1$ satisfies (ii) for $P_{ij} > \beta_1$ for any i. This means that Condition 1 is satisfied in this example when $r_0 = r_1$ and, in light of Theorem 1, no fully informative equilibria exist. A critical part of this example is that the opposite signals exactly cancel out each other, so that $P_{01} = P_{10} = 0.5$. When that happens, as can be seen from the figure, Condition 1 is satisfied for any strictly positive preference bias $b_n > 0$

[11] In the figure, we set $r_0 = r_1 = 0.75$ and $b_0 = b_1 = 0.2$.

Two features of the signal structures are at work to make this particular example work. First, in the two-signal example, the signal structures are by design "symmetric." Second, holding this symmetry, Condition 1 holds for any given $b_n > 0$ when r_1 is sufficiently close to r_0. One can then deduce from these that Condition 1 holds when the signal structures are sufficiently "symmetric" and "similar." Below, we formalize and validate this conjecture.

Definition 2 Player 0's signal structure is *symmetric* if for any $i \leq \frac{K_0}{2}$,

$$\ell_0(i) \cdot \ell_0(K_0 - i) = 1.$$

Definition 3 Player 1's signal structure is δ-*close* to Player 0's if $K_0 = K_1$ and

$$\min_{t,i} \|P(s_1 = i|t) - P(s_0 = i|t)\| < \delta.$$

Proposition 2 Consider the environment in which $P(t = 0) = P(t = 1)$ and Player 0's signal structure is symmetric. Then there exists $\bar{\delta} > 0$ such that for any $\delta < \bar{\delta}$, if Player 1's information is δ-close to Player 0's, then Condition 1 holds.

Proof: Define $i(j) = K_0 - j$. Then $P_{i(j).j} \to 0.5$ as $\delta \to 0$. Therefore we can find a suitable $\bar{\delta}$. ∎

Clearly, (i) of Condition 1 is crucial for the existence of informative equilibria. By approaching from the opposite side, we can obtain a condition under which informative equilibria always exist.

Condition 2 Both of the following conditions hold:

(i) for each $j \in S_1$, there exits no i such that $1 - \beta_0 \geq P_{ij} \geq \beta_1$, and

(ii) there exists j, i, and i' such that $P_{ij} > 1 - \beta_0$ and $P_{i'j} < \beta_1$.

Theorem 2 Fully informative equilibria always exist under Condition 2.

Proof: See Appendix.

To see this result, we again go back to the two-signal example and consider the canonical case of a totally uninformed receiver. The fact that the receiver is totally uninformed is equivalent to $r_1 = 0.5$ in the two-signal example, i.e., the signals are not at all informative. As one can easily confirm, in this setup, fully informative equilibria always exist as long as the preference biases are sufficiently small. If the sender's signal is informative even slightly, i.e., $r_0 > 0.5$, then $P_{s_0 s_1} \neq 0.5$ for any (s_0, s_1). Then, as $b_n \to 0$ and hence $\beta_n \to 0.5$, Condition 2 is bound to hold at some point. This leads to the outcome that can easily be conjectured: with discrete state and action spaces, perfect revelation of private information via cheap talk is feasible as long as the preferences are sufficiently congruent. This is illustrated in Figure 2: in the figure, we use the same parameters as in Figure 1, except that we set $r_1 = 0.5$. In this case, the corresponding posterior belief P_{ij} lies outside of either β_1 or $1 - \beta_0$ for each $j \in S_1$, so that Condition 2 is satisfied.

3.4 Hybrid equilibria

When there are more than two signals, as in our general setup, there arises the possibility of (partially pooling) hybrid equilibria where the sender bundles some of the signals into a single message. Although we do not place much emphasis on this possibility because our aim is simply to show that the receiver's prior information impedes efficient communication, rather than to characterize all possible equilibria of this game, we provide a brief remark about it and how that changes the informativeness of communication.

It may appear that when the number of signals is large, the analysis of hybrid equilibria becomes hopelessly tedious with so many different combinations of signals. The following result establishes, however, that we only need to look at a particular class of equilibria, which simplifies the analysis to some extent.

Definition 4 An equilibrium (M, A) is *partitioned* if there exists a partition $\{I_1, \ldots, I_L\}$ of S_1 with the following properties:

- there exist distinctive $m_1, \ldots, m_K$ such that $i \in I_k \Rightarrow M(i) = m_k$,
- $k < k' \Rightarrow i < i' \quad \forall i \in I_k, \ \forall i' \in I_{k'}$, and

Table 1: Player n's signal structure

	$s_n = 0$	$s_n = 1$	$s_n = 2$
$t = 1$	qr	$1 - q$	$q(1 - r)$
$t = 0$	$q(1 - r)$	$1 - q$	qr

- $k < k' \Rightarrow$ there exists some j such that $A(m_k, j) = 1,\ A(m_{k'}, j) = 0$.

Proposition 3 If there exists an equilibrium (M, A), then there also exists a corresponding partitioned equilibrium (M^*, A^*) which realizes the same allocation in the sense that $A(M(i), j) = A^*(M^*(i), j)$ for any $i \in S_0$ and $j \in S_1$.

Proof: See Appendix.

This result allows us to focus on the class of partitioned equilibria, amongst a plethora of all plausible hybrid equilibria, without loss of generality. Given this, we first show that in order to obtain an analogous result to Theorem 1 for hybrid equilibria, Condition 1 is not sufficient. This is illustrated by the following example which satisfies the condition.

- $P(t = 0) = P(t = 1) = 1/2$.
- $S_n = \{0, 1, 2\}$.
- Player n's signal structure is as depicted in Table 1, where $r \in [1/2, 1)$, $q \in (0, 1)$.

In this example, since Condition 1 holds, there exist no informative equilibria. With more than two signals, however, there may still be a way to convey information by arbitrarily combining nearby signals. To see this let the partition be given by $I_1 = \{0\}$ and $I_2 = \{1, 2\}$. This partition of the signals can credibly convey some information if and only if the following (M, A) constitutes an equilibrium:

$$A(m, j) = \begin{cases} 0 & \text{if } m = M(1) = M(2) \text{ and } j = 1, 2, \\ 1 & \text{otherwise.} \end{cases}$$

The equilibrium condition for such (M, A) is

$$\beta_0 \geq \frac{\sum_{j=1,2}(1-P_{0j})P(s_1=j|s_0=0)}{\sum_{j=1,2}P(s_1=j|s_0=0)},$$
$$\beta_1 \geq \frac{\sum_{i=1,2}P_{i1}P(s_0=i|s_1=1)}{\sum_{i=1,2}P(s_0=i|s_1=1)}.$$

These inequalities are rewritten as

$$\beta_0 \geq \frac{qr(1-r)+(1-q)(1-r)}{2qr(1-r)+1-q},$$
$$\beta_1 \geq \frac{1-q+q(1-r)}{2-q}.$$

One can easily show that there exists (r, q) satisfying the above inequalities: for example let $r = 1-\epsilon^2$ and $q = 1-\epsilon$, then under any sufficiently small $\epsilon > 0$, the above inequalities hold.

This example shows, not surprisingly, that more stringent conditions are in general needed to rule out hybrid equilibria altogether. This is due to the fact that even if the sender's signal structure is sufficiently "symmetric" to begin with, this symmetry can easily be broken by arbitrarily bundling some nearby signals into a single message. Condition 1 needs to be modified to rule out any hybrid equilibria. The following is a sufficient condition for communication to have no value at all in the current setup.

Condition 3 There exist $\hat{i} \in S_0 - \{K_0\}$ and $\hat{j} \in S_1 - \{K_1\}$ such that

$$1-\beta_0 > P_{ij} > \beta_1 \quad \text{for } i > \hat{i} \text{ and } j \leq \hat{j}, \text{ or}$$
$$i \leq \hat{i} \text{ and } j > \hat{j}.$$

Theorem 3 No information can be conveyed via cheap talk under Condition 3.

PROOF: See Appendix.

4 A case for rational ignorance

In most organizations, information is typically dispersed among different individuals, and it is hence imperative to somehow find a way to aggregate it. In particular, our results show that the distribution of information within an organization significantly affects the extent to which communication can be an effective tool to exchange information. To obtain this result, we have thus far assumed that the quality of information held by each party is exogenously given and, more importantly, publicly known. In more realistic cases, though, the quality of information can be endogenous and only privately known. In that case, the receiver might be tempted to collect information on her own, but the possibility that she may resort to that option could severely limit her ability to extract information from the sender. Unless the receiver is more adept at acquiring information than the sender by a significant margin, she might as well do nothing, rather than try to obtain imperfect information. In a situation like this, therefore, the receiver must find a way to credibly commit herself to remaining ignorant to extract any useful information from the sender.

To put this idea in context, we extend the basic setup by adding the information-acquisition stage. Suppose that at the beginning, the receiver has a chance to acquire her own information by incurring some cost d, while her choice is not observable to the sender. If the receiver chooses to do so, then the accuracy of her signal is given by $r_1 = r > 0.5$; if not, the signal contains no information, i.e., $r_1 = 0.5$. As above, the accuracy of the sender's signal is exogenously given and we let $r_0 = r$. We also assume that the preference bias is sufficiently small to satisfy $r > \beta$.

It is straightforward to verify that if d is sufficiently small, it is optimal for the receiver to acquire the information, regardless of whether the sender reveals his observation truthfully or not: that is, acquiring the information is a dominant strategy for the receiver.[12] Given this, the sender has no incentive to reveal his observation truthfully, making communication totally ineffective. The unique Nash equilibrium of this game is that the receiver acquires the information at her own cost and the sender sends an uninformative babbling message, which is clearly an inefficient outcome by the margin of the information cost d. While any

[12]If the sender conceals his observation by sending a babbling message, it is clearly optimal for the receiver to acquire her own information as long as the cost d is sufficiently small. Now suppose that the sender reveals his observation truthfully. If the receiver does not acquire the information, she simply takes an action recommended by the sender, and the expected payoff is $r + 0.5b$. If the receiver chooses to acquire the information, on the other hand, she overrules the sender's recommendation when $m = 0$ and $s_1 = 1$, and the expected payoff is $r + (0.5 + r(1-r))b$. The receiver chooses to acquire the information if and only if $r(1-r)b \geq d$, which is again satisfied when d is sufficiently small.

organization would hope to avoid this dilemma, the problem is more deep-rooted than it appears, because it is in principle much harder to prove one's ignorance.

To overcome this problem, a hierarchical organization must be designed to assure that those higher up in the hierarchy, with decision-making authority, credibly remain ignorant. This can be achieved, for instance, if there is a way to raise the cost of information acquisition in some visible way. While there are possibly many means to achieve this end, one way is to let the decision maker oversee many subordinates in order to keep her too busy to get into every single decision to be made. This is typical of most hierarchical organizations where a single manager supervises several subordinates each of whom has expertise in his own jurisdiction. A wider span of control can be seen as a visible and credible way to keep decision makers away from obtaining imperfect information from alternative sources. We argue that this aspect of our model sheds some light on the pyramidal nature of hierarchies pertaining to most organizations. Although we do not explore this issue more as it is clearly outside the scope of this paper, exploring this issue in this light may provide some new insight on the role of communication and information acquisition in organizational design.

5 Conclusion

This paper explores the extent to which cheap-talk communication can credibly convey meaningful information when the receiver is partially informed. As it turns out, the receiver's prior knowledge matters and makes a non-trivial difference in the quality of information that can be extracted from the sender. As a general rule, communication becomes less efficient as the receiver becomes more informed. This result yields a critical implication: in order to facilitate communication, it may be advisable for the receiver to refrain herself from acquiring her own information, even if it can be done with a relatively small cost.

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Appendix

Proof of Proposition 1: Here, we extend the strategy space to include mixed strategies for more generality in order to show that mixed strategies basically have no bite in the current setting. An equilibrium, in its most generic form, is defined by $(\alpha_i, \gamma_{ij}, q_{ij})_{i,j=0,1}$ where

- α_i is the probability that the sender sends $m = 1$ when he receives $s_0 = i$,
- γ_{ij} is the probability that the receiver chooses $a = 1$ when she receives $m = i$ and $s_1 = j$, and
- q_{ij} is the receiver's posterior belief over $t = 1$ when she receives $m = i$ and $s_1 = j$.

In what follows, we assume $\alpha_0 \leq \alpha_1$ without loss of generality.

Suppose that $r_0 \geq 1 - \beta$ for a while. Since

$$P_{01} = \frac{(1 - r_0)r_1}{(1 - r_0)r_1 + r_0(1 - r_1)},$$

P_{01} is strictly increasing in r_1, $P_{01}|_{r_1=0.5} = 1 - r_0 \leq \beta$, and $P_{01}|_{r_1=r_0} = 0.5 > \beta$, which implies that R is uniquely defined. Moreover, $P_{01} < \beta$ for $r_1 < R$ and $P_{01} > \beta$ for $r_1 > R$.

Next, we show that for $r_1 \in [0.5, R]$, there exists a fully informative equilibrium. To see this, it suffices to check the incentive for truth telling when $s_0 = 1$, i.e.,

$$P(s_1 = 0 \mid s_0 = 1)(\gamma_{10} - \gamma_{00})(P_{10} - 1 + \beta) \geq P(s_1 = 1 \mid s_0 = 1)(\gamma_{01} - \gamma_{11})(P_{11} - 1 + \beta). \tag{7}$$

For $r_1 \leq R$, given that the sender reveals truthfully, it is optimal (at least weakly) for the receiver to choose $\gamma_{00} = \gamma_{01} = 0$ and $\gamma_{10} = \gamma_{11} = 1$. Since $P_{11} \geq P_{10} \geq 1 - \beta$ in this range, this condition is satisfied.

We can also show that there exist no informative equilibria for $r_1 > R$. A key observation to be made in this range is that $q_{11} > q_{01} > \beta$ and hence $\gamma_{01} = \gamma_{11} = 1$ for any (α_0, α_1) (as long as $\alpha_1 > \alpha_0$). The definition of informative equilibria then requires that $\gamma_{10} > \gamma_{00}$. Given this, it is now almost straightforward to see that (7) cannot be satisfied in this range, as the condition can now be written as

$$P(s_1 = 0 \mid s_0 = 1)(\gamma_{10} - \gamma_{00})(P_{10} - 1 + \beta) \geq 0.$$

This obviously fails to hold since $P_{10} < 1 - \beta$ for $r_1 > R$.

Finally, suppose that $r_0 < 1 - \beta$. In this case $P_{10} < 1 - \beta$ for any r_1. Therefore, we can show that there exist no informative equilibria as similarly done in the previous paragraph. ∎

Proof of Lemma 1: We have

$$\begin{aligned}
&\mathbb{E}\left[u_1(t,1)|s_0 \in \mathcal{S}_0(m),\ s_1 = j\right] - \mathbb{E}\left[u_1(t,0)|s_0 \in \mathcal{S}_0(m), s_1 = j\right] \\
&= P(t=1|s_0 \in \mathcal{S}_0(m), s_1 = j)(1+b_1) - P(t=0|s_0 \in \mathcal{S}_0(m), s_1 = j)(1-b_1) \\
&= 2\left[P(t=1|s_0 \in \mathcal{S}_0(m),\ s_1 = j) - \beta_1\right],
\end{aligned}$$

where

$$\begin{aligned}
&P(t=1|s_0 \in \mathcal{S}_0(m), s_1 = j) \\
&= \frac{P(t=1, s_0 \in \mathcal{S}_0(m), s_1 = j)}{P(s_0 \in \mathcal{S}_0(m), s_1 = j)} \\
&= \frac{\sum_{i \in \mathcal{S}_0(m)} P(t=1, s_0 = i, s_1 = j)}{\sum_{i \in \mathcal{S}_0(m)} P(s_0 = i, s_1 = j)} \\
&= \frac{\sum_{i \in \mathcal{S}_0(m)} \frac{P(t=1, s_0=i, s_1=j)}{P(s_0=i, s_1=j)} P(s_0 = i, s_1 = j)}{\sum_{i \in \mathcal{S}_0(m)} P(s_0 = i, s_1 = j)} \\
&= \frac{\sum_{i \in \mathcal{S}_0(m)} P_{ij} P(s_0 = i, s_1 = j)}{\sum_{i \in \mathcal{S}_0(m)} P(s_0 = i, s_1 = j)} \\
&= \frac{\sum_{i \in \mathcal{S}_0(m)} P_{ij} P(s_0 = i|s_1 = j)}{\sum_{i \in \mathcal{S}_0(m)} P(s_0 = i|s_1 = j)}.
\end{aligned}$$

∎

Proof of Lemma 2: To prove the lemma, we first establish the following results.

Lemma 3 For any nonempty $\tilde{S}_0 \subseteq S_0$ and any $j < j'$,

$$\frac{\sum_{i \in \tilde{S}_0} P_{ij} P(s_0 = i|s_1 = j)}{\sum_{i \in \tilde{S}_0} P(s_0 = i|s_1 = j)} > \frac{\sum_{i \in \tilde{S}_0} P_{ij'} P(s_0 = i|s_1 = j')}{\sum_{i \in \tilde{S}_0} P(s_0 = i|s_1 = j')}.$$

Proof: We obtain

$$
\begin{aligned}
&\frac{\sum_{i\in\tilde{S}_0} P_{ij} P(s_0=i|s_1=j)}{\sum_{i\in\tilde{S}_0} P(s_0=i|s_1=j)} - \frac{\sum_{i\in\tilde{S}_0} P_{ij'} P(s_0=i|s_1=j')}{\sum_{i\in\tilde{S}_0} P(s_0=i|s_1=j')}\\
&= \frac{\sum_{i\in\tilde{S}_0} P_{ij} P(s_0=i,s_1=j)}{\sum_{i\in\tilde{S}_0} P(s_0=i,s_1=j)} - \frac{\sum_{i\in\tilde{S}_0} P_{ij'} P(s_0=i,s_1=j')}{\sum_{i\in\tilde{S}_0} P(s_0=i,s_1=j')}\\
&= \frac{1}{\left(\sum_{i\in\tilde{S}_0} P(s_0=i,s_1=j)\right)\left(\sum_{i\in\tilde{S}_0} P(s_0=i,s_1=j')\right)}\\
&\quad\times\Big\{\sum_{i\in\tilde{S}_0}(P_{ij}-P_{ij'})P(s_0=i,s_1=j)P(s_0=i,s_1=j')\\
&\qquad+\sum_{i,i'\in\tilde{S}_0,i<i'}\big[(P_{ij}-P_{i'j'})P(s_0=i,s_1=j)P(s_0=i',s_1=j')\\
&\qquad\quad+(P_{i'j}-P_{ij'})P(s_0=i',s_1=j)P(s_0=i,s_1=j')\big]\Big\}.
\end{aligned}
$$

It is easily verified that $P_{ij} > P_{ij'}$ by (4). Also, for any $i < i'$,

$$
\begin{aligned}
&(P_{ij}-P_{i'j'})P(s_0=i,s_1=j)P(s_0=i',s_1=j') + (P_{i'j}-P_{ij'})P(s_0=i',s_1=j)P(s_0=i,s_1=j')\\
&= P(t=0)P(t=1)P(s_1=j|t=1)P(s_1=j'|t=1)\\
&\quad\times\left[P(s_0=i|t=0)P(s_0=i'|t=1)+P(s_0=i'|t=0)P(s_0=i|t=1)\right]\left[\ell_1(j)-\ell_1(j')\right].
\end{aligned}
$$

This is positive by (4) which proves the lemma. ∎

Lemma 4 Given the receiver's strategy A satisfying (5), if $A(M(i),j)=1$ and $A(M(i'),j)=0$, then there exists no j' such that $A(M(i),j')=0$ and $A(M(i'),j')=1$.

Proof: Suppose to the contrary that there exist (i,i',j,j') such that $A(M(i),j)=1$, $A(M(i'),j)=0$, $A(M(i),j')=0$ and $A(M(i'),j')=1$. By (5),

$$
\begin{aligned}
\frac{\sum_{\tilde{i}\in\mathcal{S}_0(M(i))} P_{\tilde{i}j} P(s_0=\tilde{i}|s_1=j)}{\sum_{\tilde{i}\in\mathcal{S}_0(M(i))} P(s_0=\tilde{i}|s_1=j)} &\le \beta_1 \le \frac{\sum_{\tilde{i}\in\mathcal{S}_0(M(i'))} P_{\tilde{i}j} P(s_0=\tilde{i}|s_1=j)}{\sum_{\tilde{i}\in\mathcal{S}_0(M(i'))} P(s_0=\tilde{i}|s_1=j)},\\
\frac{\sum_{\tilde{i}\in\mathcal{S}_0(M(i))} P_{\tilde{i}j'} P(s_0=\tilde{i}|s_1=j')}{\sum_{\tilde{i}\in\mathcal{S}_0(M(i))} P(s_0=\tilde{i}|s_1=j')} &\ge \beta_1 \ge \frac{\sum_{\tilde{i}\in\mathcal{S}_0(M(i'))} P_{\tilde{i}j'} P(s_0=\tilde{i}|s_1=j')}{\sum_{\tilde{i}\in\mathcal{S}_0(M(i'))} P(s_0=\tilde{i}|s_1=j')}.
\end{aligned}
$$

Suppose $j < j'$. Then, by Lemma 3,

$$
\begin{aligned}
\beta_1 &\ge \frac{\sum_{\tilde{i}\in\mathcal{S}_0(M(i))} P_{\tilde{i}j} P(s_0=\tilde{i}|s_1=j)}{\sum_{\tilde{i}\in\mathcal{S}_0(M(i))} P(s_0=\tilde{i}|s_1=j)}\\
&> \frac{\sum_{\tilde{i}\in\mathcal{S}_0(M(i))} P_{\tilde{i}j'} P(s_0=\tilde{i}|s_1=j')}{\sum_{\tilde{i}\in\mathcal{S}_0(M(i))} P(s_0=\tilde{i}|s_1=j')}\\
&\ge \beta_1.
\end{aligned}
$$

This is a contradiction. Similarly, suppose $j > j'$. Then, Then, by Lemma 3,

$$\begin{aligned}\beta_1 &\geq \frac{\sum_{\tilde{i}\in\mathcal{S}_0(M(i'))} P_{\tilde{i}j'} P(s_0 = \tilde{i}|s_1 = j')}{\sum_{\tilde{i}\in\mathcal{S}_0(M(i'))} P(s_0 = \tilde{i}|s_1 = j')} \\ &> \frac{\sum_{\tilde{i}\in\mathcal{S}_0(M(i'))} P_{\tilde{i}j} P(s_0 = \tilde{i}|s_1 = j)}{\sum_{\tilde{i}\in\mathcal{S}_0(M(i'))} P(s_0 = \tilde{i}|s_1 = j)} \\ &\geq \beta_1.\end{aligned}$$

This is also a contradiction. ∎

We now go back to the proof of Lemma 2. Lemma 4 implies $\mathcal{S}_1(i,i') \neq \emptyset \Rightarrow \mathcal{S}_1(i',i) = \emptyset$, which directly implies the former part of the lemma.

For the latter part, first, it is clear that $\mathcal{S}_1(i,i') = \mathcal{S}_1(i',i) = \emptyset$ implies $\mathbb{E}\left[u_0(t, A(M(i),j))|s_0 = i\right] = \mathbb{E}\left[u_0(t, A(M(i'),j))|s_0 = i\right]$. Now suppose $\mathcal{S}_1(i,i') \neq \emptyset$ and $\mathcal{S}_1(i',i) = \emptyset$. Then we have

$$\begin{aligned}&\mathbb{E}\left[u_0(t, A(M(i),j))|s_0 = i\right] - \mathbb{E}\left[u_0(t, A(M(i'),j))|s_0 = i\right] \\ &= \sum_{j\in\mathcal{S}_1(i,i')} \left[P(t=1, s_1 = j|s_0 = i)(1-b_0) - P(t=0, s_1 = j|s_0 = i)(1+b_0)\right] \\ &= 2\left[\sum_{j\in\mathcal{S}_1(i,i')} P(s_1 = j|s_0 = i)\right]\left[\beta_0 - \frac{\sum_{j\in\mathcal{S}_1(i,i')} P(t=0, s_1 = j|s_0 = i)}{\sum_{j\in\mathcal{S}_1(i,i')} P(s_1 = j|s_0 = i)}\right] \\ &= 2\left[\sum_{j\in\mathcal{S}_1(i,i')} P(s_1 = j|s_0 = i)\right]\left[\beta_0 - \frac{\sum_{j\in\mathcal{S}_1(i,i')} P(t=0, s_0 = i, s_1 = j)}{\sum_{j\in\mathcal{S}_1(i,i')} P(s_0 = i, s_1 = j)}\right] \\ &= 2\left[\sum_{j\in\mathcal{S}_1(i,i')} P(s_1 = j|s_0 = i)\right]\left[\beta_0 - \frac{\sum_{j\in\mathcal{S}_1(i,i')} \frac{P(t=0,s_0=i,s_1=j)}{P(s_0=i,s_1=j)} P(s_0 = i, s_1 = j)}{\sum_{j\in\mathcal{S}_1(i,i')} P(s_0 = i, s_1 = j)}\right] \\ &= 2\left[\sum_{j\in\mathcal{S}_1(i,i')} P(s_1 = j|s_0 = i)\right]\left[\beta_0 - \frac{\sum_{j\in\mathcal{S}_1(i,i')} (1-P_{ij}) P(s_0 = i, s_1 = j)}{\sum_{j\in\mathcal{S}_1(i,i')} P(s_0 = i, s_1 = j)}\right] \\ &= 2\left[\sum_{j\in\mathcal{S}_1(i,i')} P(s_1 = j|s_0 = i)\right]\left[\beta_0 - \frac{\sum_{j\in\mathcal{S}_1(i,i')} (1-P_{ij}) P(s_1 = j|s_0 = i)}{\sum_{j\in\mathcal{S}_1(i,i')} P(s_1 = j|s_0 = i)}\right].\end{aligned}$$

Similarly, suppose $\mathcal{S}_1(i, i') = \emptyset$ and $\mathcal{S}_1(i', i) \neq \emptyset$. Then we have

$$\begin{aligned}
&\mathbb{E}\left[u_1(t, A(M(i), j))|s_0 = i\right] - \mathbb{E}\left[u_1(t, A(M(i'), j))|s_0 = i\right] \\
&= \sum_{j \in \mathcal{S}_1(i',i)} \left[P(t=0, s_1 = j|s_0 = i)(1+b_0) - P(t=1, s_1 = j|s_0 = i)(1-b_0)\right] \\
&= 2\left[\sum_{j \in \mathcal{S}_1(i',i)} P(s_1 = j|s_0 = i)\right]\left[\frac{\sum_{j \in \mathcal{S}_1(i',i)} P(t=0, s_1 = j|s_0 = i)}{\sum_{j \in \mathcal{S}_1(i',i)} P(s_1 = j|s_0 = i)} - \beta_0\right] \\
&= 2\left[\sum_{j \in \mathcal{S}_1(i',i)} P(s_1 = j|s_0 = i)\right]\left[\frac{\sum_{j \in \mathcal{S}_1(i',i)} P(t=0, s_0 = i, s_1 = j)}{\sum_{j \in \mathcal{S}_1(i',i)} P(s_0 = i, s_1 = j)} - \beta_0\right] \\
&= 2\left[\sum_{j \in \mathcal{S}_1(i',i)} P(s_1 = j|s_0 = i)\right]\left[\frac{\sum_{j \in \mathcal{S}_1(i',i)} \frac{P(t=0, s_0=i, s_1=j)}{P(s_0=i, s_1=j)} P(s_0 = i, s_1 = j)}{\sum_{j \in \mathcal{S}_1(i',i)} P(s_0 = i, s_1 = j)} - \beta_0\right] \\
&= 2\left[\sum_{j \in \mathcal{S}_1(i',i)} P(s_1 = j|s_0 = i)\right]\left[\frac{\sum_{j \in \mathcal{S}_1(i',i)} (1 - P_{ij}) P(s_0 = i, s_1 = j)}{\sum_{j \in \mathcal{S}_1(i',i)} P(s_0 = i, s_1 = j)} - \beta_0\right] \\
&= 2\left[\sum_{j \in \mathcal{S}_1(i',i)} P(s_1 = j|s_0 = i)\right]\left[\frac{\sum_{j \in \mathcal{S}_1(i',i)} (1 - P_{ij}) P(s_1 = j|s_0 = i)}{\sum_{j \in \mathcal{S}_1(i',i)} P(s_1 = j|s_0 = i)} - \beta_0\right].
\end{aligned}$$

∎

Proof of Theorem 1: Suppose to the contrary that there exists an informative equilibrium. Then $M(i) = i$ for any $i \in S_0$ and there exists some j_1 on which the sender's message is influential. We then define j^*, $\bar{i}^*$, and $\underline{i}^*$ by the following procedure:

Step 0: Go to Step 1-1 with j_1.

Step k-1: Set $\underline{i}_k = i(j_k)$ and go to Step k-2.

Step k-2: Set $\bar{i}_k = \min\{i|A(i, j_k) = 0\}$ and go to Step k-3.

Step k-3 If $j_k = \min\ \mathcal{S}_1(\underline{i}_k, \bar{i}_k)$, the set $j^* = j_k$, $\bar{i}^* = \bar{i}_k$, $\underline{i}^* = \underline{i}_k$ and stop the process. If $j_k \neq \min\ \mathcal{S}_1(\underline{i}_k, \bar{i}_k)$, then set $j_{k+1} = \min\ \mathcal{S}_1(\underline{i}_k, \bar{i}_k)$ and go to Step $(k+1)$-1.

For any k, $j_k \geq j_{k+1}$ (if any) since $j_k \in \mathcal{S}_1(\underline{i}_k, \bar{i}_k)$. Then the above procedure and j^*, $\bar{i}^*$, and $\underline{i}^*$ are well-defined. Of course, it is satisfied $\underline{i}^* = i(j^*)$ and $j^* = \min\ \mathcal{S}_1(\underline{i}^*, \bar{i}^*)$. The former fact implies that $P_{\underline{i}^* j^*} < 1 - \beta_0$ and $A(\underline{i}^*, j^*) = 1$. By the latter fact and (4), it follows that for any $j \in \mathcal{S}_1(\underline{i}^*, \overline{i}^*)$,

$$P_{\underline{i}^* j} \leq P_{\underline{i}^* j^*} < 1 - \beta_0.$$

This violates (ii) of (6). This is a contradiction. ∎

Proof of Theorem 2: Consider M that is truth telling and A that satisfies (5). Since there is no tie-breaking under Condition 2, such A is uniquely determined. Then, Lemma 1 and (ii) of Condition 2 imply that (M, A) is influential as long as it is an equilibrium.

(5) implies that for any $j \in \mathcal{S}_1(i', i)$, $P_{ij} \leq \beta_1$, which in turn implies

$$1 - P_{ij} \geq 1 - \beta_1 > \frac{1}{2} > \beta_0.$$

It follows that (iii) of (6) always holds. Then, it suffices to show that (ii) of (6) holds. Fix i and i' such that $\mathcal{S}_1(i, i') \neq \emptyset$. By (5) and (i) of Condition 2, for any $j \in \mathcal{S}_1(i, i')$, $P_{ij} > 1 - \beta_0$ holds. It follows that (ii) of (6) holds. This completes the proof. ∎

Proof of Proposition 3: (4) and the equilibrium conditions directly imply the following results (the proofs are omitted).

Lemma 5 In equilibrium, for any j, there exists $\bar{i}(j) \in \{0\} \cup S_0$ such that

$$A(M(i), j) = \begin{cases} 1 & \text{if } i \leq \bar{i}(j), \\ 0 & \text{if } i \geq \bar{i}(j) + 1. \end{cases}$$

Moreover, $j < j'$ implies $\bar{i}(j) \geq \bar{i}(j')$.

Lemma 6 Suppose that there exists an equilibrium (M, A) in which there exist i, i', m, and m' such that

$$\begin{aligned} & i < i', \\ & M(i) = m \neq m' = M(i'), \\ & A(m, j) = A(m', j) \quad \forall j. \end{aligned}$$

Then, there is also an equilibrium $(\hat{M}, A)$ such that

$$\hat{M}(\tilde{i}) = \begin{cases} m & \text{if } \tilde{i} = i', \\ M(\tilde{i}) & \text{otherwise.} \end{cases}$$

The proposition follows from these results. ∎

Proof of Theorem 3: Suppose to the contrary that there exists an equilibrium in which the sender's message matters and influences the receiver's action with positive probability. By Proposition 3, there must be an equilibrium in which there exist $i_1, i_2 \in S_0$, $j_1, j_2 \in S_1$, and $m_1, m_2 \in M$ such that

$$1 \leq i_1 < i_2 \leq K_0,$$

$$1 \leq j_1 < j_2 \leq K_1,$$

$$m_1 \neq m_2,$$

$$M(s_0) = \begin{cases} m_1 & \text{iff } 1 \leq s_0 \leq i_1, \\ m_2 & \text{iff } i_1 + 1 \leq s_0 \leq i_2, \end{cases}$$

$$A(m, s_1) = \begin{cases} 1 \quad \text{if} & m = m_1 \text{ and } 1 \leq s_1 \leq j_2, \text{ or} \\ & m = m_2 \text{ and } 1 \leq s_1 \leq j_1, \\ 0 \quad \text{if} & m = m_1 \text{ and } j_2 + 1 \leq s_1 \leq K_1, \text{ or} \\ & m = m_2 \text{ and } j_1 + 1 \leq s_1 \leq K_1. \end{cases}$$

Considering Player 1's incentive on receiving $s_1 = j_1 + 1$, (4) and Lemma 1 require that $j_1 \geq \hat{j}$ should hold. On the other hand, considering Player 0's incentive on receiving $s_0 = i_1$, (4) and Lemma 2 require that $j_1 < \hat{j}$ should hold. This is a contradiction. ∎

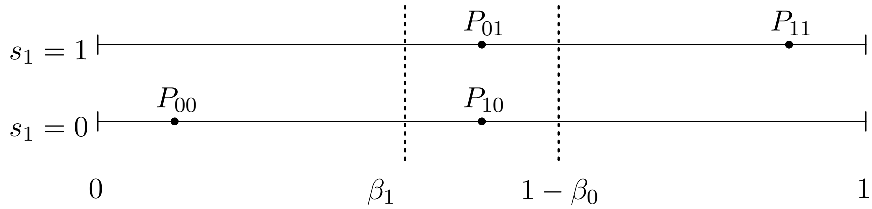

Figure 1: The signal structures that satisfy Condition 1.

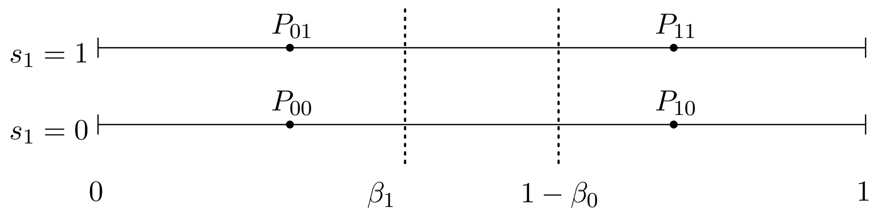

Figure 2: The signal structures that satisfy Condition 2.