Cheap Talk with an Informed Receiver

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Results

- We analyzes a cheap talk model with a partially informed sender and a partially informed receiver in discrete state space
- The more Receiver is informed $\Rightarrow$ the less incentive to tell the truth Sender has
- Receiver is equally informed as Sender $\Rightarrow$ $\nexists$ truth telling and influential eq.
- Implications to organizational issues such as allocation of authority and rational ignorance

Related Literature

- Multiple senders: Krishna and Morgan (2001), Ottaviani and Sorensen (2001), Battaglini (2002, 2004), Gerardi et al. (2009)
- Partially informed sender and receiver: Seidman (1990), Austen-Smith (1993), Prendergasdt (1993), Watson (1996), Doraszelski et al. (2003), Olszewski (2004)
- Others: Blume et al. (2007), Galeotti et al. (2009)

Timing

- Nature chooses state $\mathbf{t = 1, 2}$ with prob 1/2
- Player $\mathbf{n = 1, 2}$ receives private signal $\mathbf{s_n = 1, 2}$.
  $\mathbf{s_1}$ and $\mathbf{s_2}$ are drawn independently and

$$\mathbf{P(s_n = k|t = k) = r_n \in [1/2, 1]}$$

- Upon observing $\mathbf{s_1}$, Player 1 (Sender) sends a costless message $\mathbf{m = 1, 2}$
- Upon observing $\mathbf{s_2}$ and $\mathbf{m}$, Player 2 (Receiver) chooses an action $\mathbf{a = 1, 2}$

Payoffs

- Player $\mathbf{n}$'s payoff: $\mathbf{u_n(t, a) = \mathbb{I}(a = t) + b_n \mathbb{I}(a = n)}$
- $\beta_\mathbf{n} = \mathbf{(1 + b_n)/2}$
- We assume $\beta_\mathbf{n} \in \mathbf{(1/2, 1)} \;\Leftrightarrow\; \mathbf{b_n} \in \mathbf{(0, 1)}$
- We assume $\mathbf{r_1} \geq \mathbf{max}\{\beta_1, \beta_2\}$

Strategies, Posterior Belief, and Equilibrium

- Sender's (pure) strategy: $\mathbf{m} = \mathbf{M}(\mathbf{s}_1)$
- Receiver's (pure) strategy: $\mathbf{a} = \mathbf{A}(\mathbf{m}, \mathbf{s}_2)$
- Posterior beliefs over $\mathbf{t} = \mathbf{1}$: $\mathbf{P}_{\mathbf{ij}} = \mathbf{P}(\mathbf{t} = \mathbf{1} | \mathbf{s}_1 = \mathbf{i}, \mathbf{s}_2 = \mathbf{j})$
- Eq. is *truth telling* if $\mathbf{M}(\mathbf{i}) = \mathbf{i}\ \forall \mathbf{i}$
- Eq. is *influential* if $\exists \mathbf{j}, \mathbf{m}, \mathbf{m}'$ s.t. $\mathbf{A}(\mathbf{m}, \mathbf{j}) \neq \mathbf{A}(\mathbf{m}', \mathbf{j})$
- Eq. is *fully informative* if it is truth telling and influential

Truth Telling Equilibrium

- Receiver's best response:

$$\mathbf{P_{ij}} > \beta_2 \ \Rightarrow \mathbf{A(i,j) = 1}$$
$$\mathbf{P_{ij}} < \beta_2 \ \Rightarrow \mathbf{A(i,j) = 2}$$

- $\mathbf{r_2 = 1/2} \Rightarrow \exists$ fully informative eq.
- $\mathbf{r_2} \uparrow \mathbf{r_1} \Rightarrow \mathbf{P_{12}} \rightarrow \mathbf{1/2}$ and $\mathbf{P_{21}} \rightarrow \mathbf{1/2}$
$\Rightarrow \mathbf{A(2,2) = A(1,2) = A(2,1) = 2}$
- Eq. is influential only if $\mathbf{A(1,1) = 1}$

Truth Telling Equilibrium (cont'd)

- Sender's incentive condition for $\mathbf{M(2) = 2}$:

$$\begin{aligned} \mathbf{r_1} &\geq \mathbf{r_1 [r_2 + (1 - r_2) b_1] + (1 - r_1) r_2 (1 + b_1)} \\ &= \mathbf{r_2 + [r_1(1 - r_2) + (1 - r_1) r_2] \, b_1} \end{aligned}$$

- It does not hold when $\mathbf{r_2 \uparrow r_1}$

Setup

- We allow $\mathbf{P(t=1) \neq P(t=2)}$
- Player $\mathbf{n}$'s signal $\mathbf{S_n = \{1, \ldots, |S_n|\}}$
- $\ell_n(\mathbf{i}) = \dfrac{\mathbf{P(s_n = i | t = 1)}}{\mathbf{P(s_n = i | t = 2)}}$
- $\ell_n(1) > \ell_n(2) > \cdots > \ell_n(|\mathbf{S_n}|)$

Equilibrium Condition for Receiver (Lemma 1)

- $\mathcal{S}_1(m) = \{i \in S_1 | M(i) = m\}$
- $\dfrac{\sum_{i \in \mathcal{S}_1(m)} P_{ij} P(s_1 = i | s_2 = j)}{\sum_{i \in \mathcal{S}_1(m)} P(s_1 = i | s_2 = j)} > \beta_2 \Rightarrow A(m, j) = 1$
- $\dfrac{\sum_{i \in \mathcal{S}_1(m)} P_{ij} P(s_1 = i | s_2 = j)}{\sum_{i \in \mathcal{S}_1(m)} P(s_1 = i | s_2 = j)} < \beta_2 \Rightarrow A(m, j) = 2$

Equilibrium Condition for Sender (Lemma 2)

- $\mathcal{S}_2(i,i') = \{j \in S_2 | A(M(i),j) = 2, A(M(i'),j) = 1\}$
- $\mathbb{E}[u_1(t, A(M(i),j)) | s_1 = i] \geq \mathbb{E}[u_1(t, A(M(i'),j)) | s_1 = i]$ iff
  - $\mathcal{S}_2(i,i') = \mathcal{S}_2(i',i) = \emptyset$,
  - $\mathcal{S}_2(i,i') \neq \emptyset$ & $\dfrac{\sum_{j\in\mathcal{S}_2(i,i')} P_{ij} P(s_2 = j | s_1 = i)}{\sum_{j\in\mathcal{S}_2(i,i')} P(s_2 = j | s_1 = i)} \leq 1 - \beta_1$, or
  - $\mathcal{S}_2(i',i) \neq \emptyset$ & $\dfrac{\sum_{j\in\mathcal{S}_2(i',i)} P_{ij} P(s_2 = j | s_1 = i)}{\sum_{j\in\mathcal{S}_2(i',i)} P(s_2 = j | s_1 = i)} \geq 1 - \beta_1$

Impossibility Theorem (Theorem 1)

- Condition 1: For any $\mathbf{j}$,
  - $\exists \mathbf{i(j)}$ s.t. $\mathbf{1} - \beta_1 < \mathbf{P}_{\mathbf{i(j).j}} < \beta_2$,
  - $\mathbf{P}_{\mathbf{ij}} > \beta_2 \ \forall \mathbf{i}$, or
  - $\mathbf{P}_{\mathbf{ij}} < \beta_2 \ \forall \mathbf{i}$
- Theorem 1: Condition 1 $\Rightarrow \nexists$ fully informative eq.

Sufficient Condition for Condition 1 (Proposition 1)

- Player 1's signal strucure is *symmetric* if for any $\mathbf{i} \leq (|\mathbf{S_1}| + 1)/2$, $\ell_1(\mathbf{i})\ell_1(|\mathbf{S_1}| + 1 - \mathbf{i}) = 1$
- Player 2's signal strucure is δ-*close* to Player 1's if $|\mathbf{S_1}| = |\mathbf{S_2}|$ and $\min_{\mathbf{t},\mathbf{i}} \|\mathbf{P}(\mathbf{s_2} = \mathbf{i}|\mathbf{t}) - \mathbf{P}(\mathbf{s_1} = \mathbf{i}|\mathbf{t})\| < \delta$
- Proposition 1: If $\mathbf{P}(\mathbf{t} = 1) = \mathbf{P}(\mathbf{t} = 2)$, Player 1's signal structure is symmetric, $\exists \bar{\delta} > \mathbf{0}$ s.t. for any $\delta \in (\mathbf{0}, \bar{\delta})$, if Player 2's signal structure is δ-close to Player 1's, then Condition 1 holds

Possibility Theorem (Theorem 2)

- Condition 2:
  - $\forall \mathbf{j}$, $\nexists \mathbf{i}$ s.t. $\mathbf{1} - \boldsymbol{\beta}_1 \leq \mathbf{P}_{\mathbf{ij}} \leq \boldsymbol{\beta}_2$, and
  - $\exists \mathbf{j}, \mathbf{i}, \mathbf{i}'$ s.t. $\mathbf{P}_{\mathbf{ij}} < \mathbf{1} - \boldsymbol{\beta}_1$ and $\mathbf{P}_{\mathbf{i}'\mathbf{j}} > \boldsymbol{\beta}_2$
- Theorem 2: Condition 2 $\Rightarrow$ $\exists$ fully informative eq.

Characterization of Hybrid Equilibria

- Proposition 3: If there exists an equilibrium, then there exits a corresponding *partitioned* equilibrium which is outcome equivalent

Example: No fully informative equilibrium

s_1 / s_2	1	2	3
1			1/2
2		1/2	
3	1/2		

Example: Not Truth Telling But Influential Equilibrium

s_1 / s_2	1	2	3
$s_1 \longrightarrow m_1$			
$s_2 \longrightarrow m_1$		(1/2)	
$s_3 \longrightarrow m_3$	(1/2)		

Impossibility Theorem (Theorem 3)

- Condition 3: $\exists \hat{\mathbf{i}} \in \mathbf{S_1} - \{|\mathbf{S_1}|\}$ and $\exists \hat{\mathbf{j}} \in \mathbf{S_2} - \{|\mathbf{S_2}|\}$ s.t.

$$\beta_2 > \mathbf{P_{ij}} > \mathbf{1} - \beta_1$$
$$\mathbf{i} > \hat{\mathbf{i}}, \mathbf{j} \leq \hat{\mathbf{j}}$$
$$\mathbf{i} \leq \hat{\mathbf{i}}, \mathbf{j} > \hat{\mathbf{j}}$$

- Theorem 3: Condition 3 $\Rightarrow \nexists$ influential eq.

Extended Model

- $\mathbf{h} \in \{0, 1\}$: investment
- $\mathbf{u_2(t, a, h) = V(h)\mathbb{I}(a = t) + b_2\mathbb{I}(a = 2) - ch}$
- $\mathbf{V(1) = v > V(0) = 1}$
- $\mathbf{v\bar{p} - c = \bar{p} \Leftrightarrow \bar{p} = c/(v - 1)}$
- We assume $\mathbf{P_{11} \geq \bar{p} > \max\{r_1, r_2\}}$
  $\Rightarrow$ combination of both players' signals is important

Allocation of Authority

- Assume $\beta_1 \leq \mathbf{P}_{12} \leq \beta_2 \leq \mathbf{P}_{11}$
- Player 2 $=$ DM (Receiver) $\Rightarrow$ both players' signals are exploited
- Player 1 $=$ DM (Receiver) $\Rightarrow$ no cheap talk information
- Less informed player sholuld be DM

Rational Ignorance

- After receiving $\mathbf{m}$, Receiver chooses to acquire information (i.e., receiving $\mathbf{r_2}$) with cost $\mathbf{d}$ or not
- $\mathbf{d}$: small
  $\Rightarrow$ Receiver chooses to acquire information on receiving $\mathbf{m = 1}$
  $\Rightarrow$ No incentive for Sender to tell the truth
- Receiver benefits if she can commit to not acquiring information by herself

Future Research

- Full characterization and welfare analysis
- Model with continuous state space
- More explicit model of information aquisition