On the Role of Tax-Subsidy Scheme in Money Search Models

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Motivation

- Real indeterminacy of stationary equilibria in models with divisible money: Green and Zhou (1998), Zhou (1999), Matsui and Shimizu (2005), Kamiya and Shimizu (2006)
- Policy implication was obtained by comparative statics method in the literature
- Indeterminacy result nullifies comparative statics method
- Reconsider policy

Basic Logic

- Source of indeterminacy = 1 degree of freedom in stationary condition (Kamiya and Shimizu 2006)
- Tax-subsidy policies remove it

Results

- Consider some type of tax-subsidy schemes
- Some tax-subsity scheme makes a stationary equilirbium locally determinate
- Moreover, almost every stationary equilibrium without tax-subsidy scheme can be approximated by some appropriately designed scheme
- Size of tax-subsidy scheme can be arbitrarily small

Basic Model

- Zhou (1999) model
- Time: continuous
- A continuum of infinitely lived agents with measure $\mathbf{1}$
- $\mathbf{k}$ types of agents with equal fractions
- $\boldsymbol{\kappa} = \mathbf{1}/\mathbf{k}$
- $\mathbf{k}$ types of commodity goods: non-durable
- type $\mathbf{i}$ agent can produce one unit of good $\mathbf{i+1}$ with cost $\mathbf{c > 0}$
- type $\mathbf{i}$ agent obtains $\mathbf{u}$ ($> \mathbf{c}$) only if she consumes one unit of good $\mathbf{i}$

Basic Model (cont'd)

- Money: durable and divisible
- Pairwise random matching: Poisson process with μ
- Matching between type $\mathbf{i}$ and $\mathbf{i+1}$
- $\mathbf{i}$: seller, $\mathbf{i+1}$: buyer
- Bargaining procedure: seller's take-it-or-leave-it offer, ignorant of buyer's money holdings
- γ: discount rate

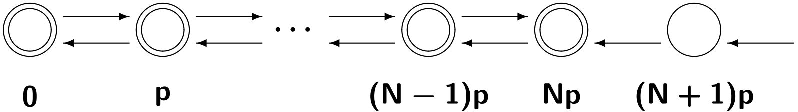
Single-Price Equilibrium with Price $\mathbf{p}$

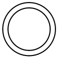
- All trades occur with price $\mathbf{p}$
- A seller with $\mathbf{np}$ for $\mathbf{n} < \mathbf{N}$ offer price $\mathbf{p}$
- A seller with $\mathbf{np}$ for $\mathbf{n} \geq \mathbf{N}$ rejects a sale
- A buyer with $\mathbf{np}$ for $\mathbf{n} > \mathbf{0}$ accepts an offer $\mathbf{p}$

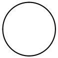
Money Holdings Distribution

- $\mathbf{h_n}$: measure of agents with $\mathbf{np}$ units of money
- $\mathbf{h} = (\mathbf{h_0}, \mathbf{h_1}, \ldots, \mathbf{h_N})$: distribution of money holdings
- $\mathbf{N} < \infty$: endogenously determined upper bound of money holdings
- $\mathbf{1} - \mathbf{h_0}$: measure of potential buyers
- $\mathbf{1} - \mathbf{h_N}$: measure of potential sellers

Transition of Money Holdings



 : Money holdings on the equililbrium

 : Money holdings off the equilibrium

Money Holdings Distribution (cont'd)

- $\mathbf{M}$: exogenously given supply of money
- $\mathbf{h}$ is determined $\Rightarrow$ $\mathbf{p}$ is determined s.t. $\sum_{\mathbf{n}=0}^{\mathbf{N}} \mathbf{p n h_n} = \mathbf{M}$
- We can ignore $\mathbf{p}$ (as long as $\mathbf{h_0} \neq \mathbf{1}$)

Modeling "Policy"

- Introducing "policy" into this framework
- $\Rightarrow$ following Aiyagari et al. (1996) approach
- Government agents are
  - involeved in random matching process, like private agents
  - programmed to behave accordingly to a set of instructions
  - granted the right to collect tax from or give subsidy to the agents they are matched with
- $\mathbf{G}$: measure of government agents
- $\phi = \frac{(1+\mathbf{G})\gamma}{\mu\kappa}$: degree of friction

Modeling "Policy" (cont'd)

- A tax-subsidy scheme is a set of instructions with the form $\mathbf{t} = (\mathbf{t_0}, \mathbf{t_1}, \ldots, \mathbf{t_N})$:
  - $\mathbf{t_n} > \mathbf{0}$: instruction to give subsidy $\mathbf{p}$ to agetns with $\mathbf{np}$ with probability $\mathbf{t_n}$
  - $\mathbf{t_n} < \mathbf{0}$: instruction to collect tax $\mathbf{p}$ from agetns with $\mathbf{np}$ with probability $-\mathbf{t_n}$
  - $\mathbf{t_0} \geq \mathbf{0}$, $\mathbf{t_N} \leq \mathbf{0}$
- $\mathbf{t} = \mathbf{0}$: case without tax-subsidy
- $\mathbf{t} \neq \mathbf{0}$: case with tax-subsidy

SPEs without Tax-Subsidy

Theorem 1 [Kamiya et al. (2005)]
For any positive integer **N**, if

$$(\phi+1)^{\mathbf{N}} < \frac{\mathbf{u}}{\mathbf{c}} < \frac{\phi(\phi+1)^{2\mathbf{N}}}{(\phi+1)^{\mathbf{N}}-1},$$

then there is a continuum of SPEs with upper bound **N**
in which all incentive conditions are satisfied with strict inequalities

Stationary Condition without Tax-Subsidy

SPE with $\mathbf{N=2}$

$$\mathbf{h_0 + h_1 + h_2 = 1}$$
$$\mathbf{h_1(1-h_2) = h_0(1-h_0)}$$
$$\mathbf{h_0(1-h_0) + h_2(1-h_2) = h_1(1-h_0+1-h_2)}$$
$$\mathbf{h_1(1-h_0) = h_2(1-h_2)}$$
$$\mathbf{Inflow} \times \left(\frac{\mathbf{1+G}}{\boldsymbol{\mu\kappa}}\right) = \mathbf{Outflow} \times \left(\frac{\mathbf{1+G}}{\boldsymbol{\mu\kappa}}\right)$$

Stationary Condition without Tax-Subsidy (cont'd)

- $\Rightarrow (\mathbf{h}_0, \mathbf{h}_1, \mathbf{h}_2) = \left(\mathbf{h}_0, \frac{-\mathbf{h}_0+\sqrt{\mathbf{h}_0(4-3\mathbf{h}_0)}}{2}, \frac{2-\mathbf{h}_0-\sqrt{\mathbf{h}_0(4-3\mathbf{h}_0)}}{2}\right)$
- Source of real indeterminacy
- $(\mathbf{h}_0, \mathbf{h}_1, \mathbf{h}_2) = (\mathbf{1/3}, \mathbf{1/3}, \mathbf{1/3})$ is the most efficient equilibrium among SPEs with $\mathbf{N} = \mathbf{2}$ (if incentive conditions are satisfied)

General Logic of Indeterminacy [Kamiya and Shimizu (2006)]

- $\mathbf{D_n} = \mathbf{I_n}$ (Inflow) $-$ $\mathbf{O_n}$ (Outflow)
- Variables: $\mathbf{N+1}$
- Equations: $\mathbf{N+2}$
- Identities: $\mathbf{2}$
  - $\sum_{n=0}^{N} D_n = 0$
  - $\sum_{n=0}^{N} nD_n = 0$
- $\Rightarrow$ 1 degree of freedom for the determination of $\mathbf{h}$

Interpretation of $\sum_{n=0}^{N} nD_n = 0$

- $\Rightarrow \sum_{n=0}^{N} npO_n = \sum_{n=0}^{N} npI_n$
- LHS: total amount of money held by the agents involved in trading before trading
- RHS: total amount of money held by the agents involved in trading after trading

SPEs with Tax-Subsidy

Consider the following policy:

- Tax-subsidy scheme: $\mathbf{t} = \epsilon \boldsymbol{\tau}$ where
  - $\boldsymbol{\tau} = (\tau_0, \tau_1, \tau_2)$
  - $\boldsymbol{\tau} \neq \mathbf{0}$
  - $\epsilon > 0$
- print money in case of budget deficit
- absorb money in case of budget surplus

Stationary Condition with Tax-Subsidy

Case 1: $\tau_1 \geq 0$

$$\tilde{h}_0 + \tilde{h}_1 + \tilde{h}_2 = 1$$

$$\tilde{h}_1(1 - \tilde{h}_2) = \tilde{h}_0(1 - \tilde{h}_0) + \underline{\tilde{h}_0 k G \tau_0 \epsilon}$$

$$\tilde{h}_0(1 - \tilde{h}_0) + \tilde{h}_2(1 - \tilde{h}_2) + \underline{\tilde{h}_0 k G \tau_0 \epsilon} - \underline{\tilde{h}_2 k G \tau_2 \epsilon}$$
$$= \tilde{h}_1(1 - \tilde{h}_0 + 1 - \tilde{h}_2) + \tilde{h}_1 k G \tau_1 \epsilon$$

$$\tilde{h}_1(1 - \tilde{h}_0) + \underline{\tilde{h}_1 k G \tau_1 \epsilon} = \tilde{h}_2(1 - \tilde{h}_2) - \underline{\tilde{h}_2 k G \tau_2 \epsilon}$$

$$\textbf{Inflow} \times \left(\frac{1+G}{\mu\kappa}\right) = \textbf{Outflow} \times \left(\frac{1+G}{\mu\kappa}\right)$$

Stationary Condition with Tax-Subsidy (cont'd)

Case 2: $\tau_1 \leq 0$

$$\tilde{h}_0 + \tilde{h}_1 + \tilde{h}_2 = 1$$

$$\tilde{h}_1(1-\tilde{h}_2) - \underline{\tilde{h}_1 k G \tau_1 \epsilon} = \tilde{h}_0(1-\tilde{h}_0) + \underline{\tilde{h}_0 k G \tau_0 \epsilon}$$

$$\tilde{h}_0(1-\tilde{h}_0) + \tilde{h}_2(1-\tilde{h}_2) + \underline{\tilde{h}_0 k G \tau_0 \epsilon} - \underline{\tilde{h}_2 k G \tau_2 \epsilon} = \tilde{h}_1(1-\tilde{h}_0 + 1 - \tilde{h}_2) - \underline{\tilde{h}_1 k G \tau_1 \epsilon}$$

$$\tilde{h}_1(1-\tilde{h}_0) = \tilde{h}_2(1-\tilde{h}_2) - \underline{\tilde{h}_2 k G \tau_2 \epsilon}$$

$$\text{Inflow} \times \left(\frac{1+G}{\mu\kappa}\right) = \text{Outflow} \times \left(\frac{1+G}{\mu\kappa}\right)$$

Stationary Condition with Tax-Subsidy (cont'd)

- $\tilde{\mathbf{h}}$ is uniquely solved such that
  - $\tilde{\mathbf{h}} \longrightarrow \mathbf{h}^*$ as $\epsilon \longrightarrow \mathbf{0}$
  - $\mathbf{h}^*$ satisfies the stationary condition without tax-subsidy
  - $\mathbf{h}^* \cdot \boldsymbol{\tau} = \mathbf{0}$
  - Therefore for $\tau_0 = -\tau_2$ and $\tau_1 = 0$, $\mathbf{h}^* = (1/3, 1/3, 1/3)$
- All incentive conditions are still satisfied if
  - $\mathbf{h}^*$ satisfies all incentive conditions with strict inequalities
  - ϵ is sufficiently small

General Logic of Deteminacy

$$[\text{Budget deficit}] = \frac{\mu \mathbf{G} \epsilon}{1+\mathbf{G}} \tilde{\mathbf{h}} \cdot \boldsymbol{\tau}$$
$$= \sum_{n=0}^{N} n \tilde{D}_n$$

- This is zero only on the stationary distribution
- Stationary distribution approaches $\mathbf{h}^*$ orthogonal to $\boldsymbol{\tau}$ as $\epsilon \longrightarrow \mathbf{0}$ (since regularity holds in Zhou model)
- Budget balancing is achieved on the equilibrium path

General Result of Basic Model

Theorem 2

For any SPE without tax-subsidy satisfying strict incentive conditions and $\delta > \mathbf{0}$, $\exists$ tax-subsidy scheme such that

- SPE with tax-subsidy is locally determinate, and
- it lies within δ-neighborhood of the SPE without tax-subsidy

Budget Balnacing

- Budget balancing rule is required
- $\Rightarrow$ One of policy variables $(\mathbf{t_0}, \ldots, \mathbf{t_N})$ turns to be an endogenous variable
- $\Rightarrow$ $\mathbf{1}$ degree of freedom comes back

General Class of Models

- No assumption on commodity goods (divisibility, cost, utility...)
- Consider a class of bargaing procedures including
  - sellers' take-it-or-leave-it offer (Zhou 1999)
  - buyers' tak-it-or-leave-it offer (Camera and Corbae 1999)
  - Nash bargaining solution (Trejos and Wright 1995)
- All trades occur with integer multiples of $\mathbf{p} > \mathbf{0}$ (equilibria with multiple prices are allowed)
- $\mathbf{N} < \infty$: determined exogenously or endogenously

General Class of Models (cont'd)

- $\mathbf{A_n = \{a_{n1}, \dots, a_{ns_n}\}}$: set of actions for an agent with $\mathbf{n}$
- **In today's talk** we focus on pure strategy equilibria:
  $\mathbf{a = (a_0, \dots, a_N)}$
- $\alpha(\mathbf{a}) = \{(\mathbf{n}, \mathbf{j}) | \mathbf{a_n = a_{nj}}\}$
- $\theta \in \mathbf{R^L}$: parameters
- Pairwise random matching with Poisson process
- Transition of money holdings: if $\mathbf{(n, a_{nj})}$ is matched with $\mathbf{(n', a_{n'j'})}$,
  $\mathbf{n \to n + f(n, j; n', j') \le N}$
  $\mathbf{n' \to n' - f(n, j; n', j') \ge 0}$

Stationary Equilibria without Tax-Subsidy

Review of Kamiya and Shimizu (2006)
Theorem 3 Given any **a**,

$$\sum_{n=0}^{N} nD_n(h, a, \theta) = 0$$

is an identity
$\Rightarrow$ 1 degree of freedom for the determination of **h** in the stationary condition

Stationary Equilibria without Tax-Subsidy (cont'd)

- $\mathbf{V_n}$: value of state $\mathbf{n}$
- $\mathbf{V = (V_0, \ldots, V_N)}$
- $\mathbf{x = (h, V, a)}$
- $\mathbf{W_{nj}(x; \theta, t)}$: value of action $\mathbf{a_{nj}}$ at state $\mathbf{n}$

Stationary Equilibria without Tax-Subsidy (cont'd)

Definition 1 $\mathbf{x} = (\mathbf{h}, \mathbf{V}, \mathbf{a})$: stationary equilibrium without tax-subsidy for $\theta \Leftrightarrow$

$$\begin{aligned}
\mathbf{D_n}(\mathbf{h}, \mathbf{a}, \theta) &= 0 & \mathbf{n} = 2, \ldots, \mathbf{N} \\
\sum_{\mathbf{n}=0}^{\mathbf{N}} \mathbf{h_n} - 1 &= 0 & \\
\mathbf{V_n} - \mathbf{W_{nj}}(\mathbf{x}; \theta, 0) &= 0 & \text{if } (\mathbf{n}, \mathbf{j}) \in \alpha(\mathbf{a}) \\
\mathbf{V_n} - \mathbf{W_{nj}}(\mathbf{x}; \theta, 0) &\geq 0 & \text{if } (\mathbf{n}, \mathbf{j}) \notin \alpha(\mathbf{a})
\end{aligned}$$

Denote LHSs by $\mathbf{g^a}(\mathbf{h}, \mathbf{V}, \theta)$

Stationary Equilibria without Tax-Subsidy (cont'd)

Additional conditions:
(i) $\mathbf{h_0 < 1}$ ($\Rightarrow$ the existence of $\mathbf{p > 0}$ s.t. $\sum_{\mathbf{n=0}}^{\mathbf{N}} \mathbf{pnh_n = M}$)
(ii) the incentive not to choose an action out of our action space
(iii) the existence of strategies at state $\eta \notin \{\mathbf{0, p, \ldots, Np}\}$ consistent with the stationary quasi-equilibrium

Stationary Equilibria without Tax-Subsidy (cont'd)

Kamiya and Shimizu (2002):
(ii) & (iii) are satisfied in some class of models including

- Zhou (1999)
- a divisivle money version of Camera and Corbae (1999)
- a divisible money version of Trejos and Wright (1995)

Stationary Equilibria without Tax-Subsidy (cont'd)

$\mathbf{E}^{\mathbf{a}}_{\theta}$: set of stationary equilibria for **a** and θ

$$\mathbf{E}^{\mathbf{a}}_{\theta} = (\mathbf{g}^{\mathbf{a}}(\cdot,\theta))^{-1}(\underbrace{\{0\} \times \cdots \times \{0\}}_{2N+1} \times \underbrace{\mathbb{R}_+ \times \cdots \times \mathbb{R}_+}_{S-N-1})$$

where $\mathbf{S} = \sum_{n=0}^{N} s_n$

Stationary Equilibria without Tax-Subsidy (cont'd)

$$\mathbf{C^a} = \underbrace{\{0\} \times \cdots \times \{0\}}_{\mathbf{2N+1}} \times \underbrace{\mathbb{R}_{++} \times \cdots \times |\mathbb{R}_{++}}_{\mathbf{S-N-1}}$$

$$\mathbf{C^{a(n,j)}} = \underbrace{\{0\} \times \cdots \times \{0\}}_{\mathbf{2N+1}} \times \underbrace{\mathbb{R}_{++} \times \cdots \times \mathbb{R}_{++} \times \{0\} \times \mathbb{R}_{++} \times \cdots \times \mathbb{R}_{++}}_{\mathbf{S-N-1}}$$

for $\mathbf{(n,j) \notin \alpha(a)}$

$$\mathbf{C^{a(n,j)(n',j')}} =$$

$$\underbrace{\{0\} \times \cdots \times \{0\}}_{\mathbf{2N+1}} \times \underbrace{\mathbb{R} \times \cdots \times \mathbb{R} \times \{0\} \times \mathbb{R} \times \cdots \times \mathbb{R} \times \{0\} \times \mathbb{R} \times \cdots \times \mathbb{R}}_{\mathbf{S-N-1}}$$

for $\mathbf{(n,j), (n',j') \notin \alpha(a)}$, and $\mathbf{(n,j) \neq (n',j')}$

Stationary Equilibria without Tax-Subsidy (cont'd)

$$\mathbf{E}^{\mathbf{a}}_{\theta} \subset (\mathbf{g}^{\mathbf{a}}(\cdot,\theta))^{-1}\left(\mathbf{C}^{\mathbf{a}} \cup (\cup_{(\mathbf{n},\mathbf{j})}\mathbf{C}^{\mathbf{a}(\mathbf{n},\mathbf{j})}) \cup (\cup_{(\mathbf{n},\mathbf{j}),(\mathbf{n}',\mathbf{j}')}\mathbf{C}^{\mathbf{a}(\mathbf{n},\mathbf{j})(\mathbf{n}',\mathbf{j}')})\right)$$

$$\mathbf{E}^{\mathbf{a}}_{\theta} \supset (\mathbf{g}^{\mathbf{a}}(\dot{;}\theta))^{-1}(\mathbf{C}^{\mathbf{a}} \cup (\cup_{(\mathbf{n},\mathbf{j})}\mathbf{C}^{\mathbf{a}(\mathbf{n},\mathbf{j})}))$$

Stationary Equilibria without Tax-Subsidy (cont'd)

Assumption 1[Regurality Condition]

- $\mathbf{g^a}$: $\mathbf{C^2}$
- $\mathbf{g^a}$ is transversal to $\mathbf{C^a}$, $\mathbf{C^{a(n,j)}}$, $\mathbf{C^{a(n,j)(n',j')}}$ for all $\mathbf{(n,j)} \notin \alpha\mathbf{(a)}$, $\mathbf{(n',j')} \notin \alpha\mathbf{(a)}$

Assumption 2[Existence Condition] $\exists\boldsymbol{\Theta}$ such that

- $\boldsymbol{\Theta}$: $\mathbf{C^2}$ manifold without boundary
- $\mathbf{E^a_\theta} \neq \emptyset \quad \forall\theta \in \boldsymbol{\Theta}$

Stationary Equilibria without Tax-Subsidy (cont'd)

Theorem 4 Under [Regurality Condition] and [Existence Condition], $\mathbf{E}^{\mathbf{a}}_{\boldsymbol{\theta}}$ is a one-dimensional manifold for almost every $\boldsymbol{\theta} \in \boldsymbol{\Theta}$. Moreover,

- at the end points of the manifold, only one of the incentive conditions is binding
- at the other points, no incentive condition is binding

Staionary Equilibria with Tax-Subsidy

$$\tilde{I}_n = I_n + \frac{\mu G}{1+G}\left(t^+_{n-1}\tilde{h}_{n-1} + t^-_{n+1}\tilde{h}_{n+1}\right),$$

$$\tilde{O}_n = O_n + \frac{\mu G}{1+G}|t_n|\tilde{h}_n,$$

where

- $t^+_n = \max\{0, t_n\}$
- $t^-_n = -\min\{0, t_n\}$
- $t_{-1} = t_{N+1} = 0$

Staionary Equilibria with Tax-Subsidy (cont'd)

Theorem 5 Given $\mathbf{a}$, if Jacobian matrix of

$$\left(\tilde{\mathbf{D}}_1, \ldots, \tilde{\mathbf{D}}_N, \sum_{n=0}^{N} \tilde{\mathbf{h}}_n - 1\right)^T$$

w.r.t. $\tilde{\mathbf{h}}$ is full rank at a stationary distribution, then it is locally unique

Staionary Equilibria with Tax-Subsidy (cont'd)

Fix $\mathbf{x^* = (h^*, V^*, a^*)}$ which is a relative interior point of the equilibrium manifold without tax-subsidy

Lemma 1 $\exists \tau$ such that

(a) $\tau \neq \mathbf{(0, \dots, 0)}$

(b) $\left(\frac{\partial \mathbf{D_n(h^*, a^*; \theta)}}{\partial \mathbf{h_i}} \right)_{\mathbf{i=0,\dots,N}} \cdot \tau = \mathbf{0}$ for $\mathbf{n = 2, \dots, N}$

(c) $\mathbf{h^*} \cdot \tau = \mathbf{0}$

Assumption 3 In addition to (a), (b), (c), such τ satisfies

(d) $\tau_{\mathbf{N}} \leq \mathbf{0}$

(e) $\tau_{\mathbf{0}} \geq \mathbf{0}$

We set $\mathbf{t} = \epsilon \tau$ for $\epsilon > \mathbf{0}$

Assumption 4 $\mathbf{\tilde{W}_{nj}}$: $\mathbf{C^2}$ w.r.t. ϵ

Staionary Equilibria with Tax-Subsidy (cont'd)

$$\tilde{g}_{\epsilon}^{a^*}(\tilde{h}, \tilde{V}) = \begin{cases} \tilde{D}_n(\tilde{h}, a^*; \theta, \epsilon\tau) & n = 1, \ldots, N \\ \sum_{n=0}^{N} \tilde{h}_n - 1 & \\ \tilde{V}_n - \tilde{W}_{nj}(\tilde{h}, \tilde{V}, a^*, \theta, \epsilon\tau) & (n, j) \in \alpha(a^*) \end{cases}$$

$$\hat{g}_{\epsilon}^{a^*}(\tilde{h}, \tilde{V}) = \begin{cases} \tilde{h} \cdot \tau & \\ \tilde{D}_n(\tilde{h}, a^*; \theta, \epsilon\tau) & n = 2, \ldots, N \\ \sum_{n=0}^{N} \tilde{h}_n - 1 & \\ \tilde{V}_n - \tilde{W}_{nj}(\tilde{h}, \tilde{V}, a^*, \theta, \epsilon\tau) & (n, j) \in \alpha(a^*) \end{cases}$$

Staionary Equilibria with Tax-Subsidy (cont'd)

For sufficiently small $\epsilon > \mathbf{0}$,
[Set of stationary equilibria for $\mathbf{a}^*$ and θ]
$=$ [Solution set of $\tilde{\mathbf{g}}_{\epsilon}^{\mathbf{a}^*} = \mathbf{0}$]
$=$ [Solution set of $\hat{\mathbf{g}}_{\epsilon}^{\mathbf{a}^*} = \mathbf{0}$]

$$\because \quad \frac{\mu \mathbf{G} \epsilon}{\mathbf{1} + \mathbf{G}} \, \tilde{\mathbf{h}} \cdot \tau = - \sum_{\mathbf{n}=\mathbf{0}}^{\mathbf{N}} \mathbf{n} \tilde{\mathbf{D}}_{\mathbf{n}}$$

Staionary Equilibria with Tax-Subsidy (cont'd)

Lemma 2 Under [Regularity Condition], Assumption 3, Assumption 4, Jacobian matrix of $\mathbf{\hat{g}_0^{a}}^*$ w.r.t. $(\tilde{\mathbf{h}}, \tilde{\mathbf{V}})$ at $(\mathbf{h}^*, \mathbf{V}^*)$ is full rank

Staionary Equilibria with Tax-Subsidy (cont'd)

Theorem 6 Under

- Regularity Condition,
- Existence Condition,
- Assumption 3,
- Assumption 4,

for almost every $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, almost every $(\mathbf{h}^*, \mathbf{V}^*) \in \mathbf{E}_{\boldsymbol{\theta}}^{\mathbf{a}^*}$, and any $\boldsymbol{\delta}$-neighborhood of $\mathbf{x}^*$, there exists a tax-subsidy scheme such that a stationary equilibrium is locally determinat and lies in the neighborhood

Sketch of Proof of Lemma 2

$$\begin{bmatrix} \begin{bmatrix} \tau_0 & \cdots & \tau_N \\ \frac{\partial D_2}{\partial h_0} & \cdots & \frac{\partial D_2}{\partial h_N} \\ & \cdots & \\ \frac{\partial D_N}{\partial h_0} & \cdots & \frac{\partial D_N}{\partial h_N} \\ 1 & \cdots & 1 \end{bmatrix} & \mathbf{0} \\ \cdots & \begin{bmatrix} J_V(V_0 - W_{0j(0)}) \\ \cdots \\ J_V(V_N - W_{Nj(N)}) \end{bmatrix} \end{bmatrix}$$