Cheap Talk with an Exit Option: A Model of Exit and Voice[*]

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Abstract

The paper presents a formal model of the exit and voice framework proposed by Hirschman [14]. More specifically, we modify Crawford and Sobel's [9] cheap talk model such that the sender of a cheap talk message has an exit option. We demonstrate that the existence of the exit option may increase the informativeness of cheap talk and improve welfare if the exit option is relatively attractive to the sender. Moreover, it is verified that perfect information transmission can be approximated in the limit. The results suggest that the exit reinforces the voice in that the credibility of the exit increases the informativeness of the voice.

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1 Introduction

Since the publication of the book *Exit, Voice, and Loyalty* by Hirschman [14], the exit-voice perspective has been widely adopted by studies in the field of political science, and it has also been extended to various studies on relationships and organizations, such as employer-employee (or union) relationships, buyer-seller relationships, hierarchies, public services, political parties, families, and adolescent development (See Hirschman [14] and [15]).

Broadly speaking, the exit and the voice are alternative means of dealing with problems that arise within an ongoing relationship or organization. For example, consider an employer-employee relationship.[1] Suppose an employee finds himself/herself in undesirable situations regarding the conditions of employment, compensation packages, and rules at the workplace. In this situation, the employee usually has two options. One is to quit the job; this is the exit option. The other is to express his dissatisfaction directly to the employer; this is the voice option. Hirschman insists that the voice as well as the exit option is important for the sustainability of relationships and organizations—a concept that has been hitherto neglected in economics.

With regard to the workings of the exit and voice options in a real economy, how the exit interacts with the voice is a point of considerable interest, which is the main point of Hirschman's discussion. From one perspective, the exit works as a complement to the voice. Indeed, in Palgrave's dictionary [15], Hirschman briefly points out, "[t]he availability and threat of exit on the part of important customer or group of members may powerfully reinforce their voice."[2] However, it is not very clear why and how the exit can reinforce the voice. The present paper aims to clarify this by analyzing a formal model of exit and voice.

In this paper, the exit is regarded as a decision to terminate an ongoing relationship, whereas the voice is interpreted as an activity involving sending a costless message that serves for the improvement of the relationship. In other words, we identify the voice with "cheap talk" for transmitting useful information.[3] Among others, the model by Crawford

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[1] See Freeman [12].

[2] In his 1970 book [14], Hirschman seems to emphasize on a substitute aspect between exit and voice. However, in 1987 Palgrave's dictionary [15], he turned to insist that a complementarity aspect of exit and voice is also important.

[3] As we see later, Banerjee and Somanathan [2] also identify voice as an activity of sending a cheap talk

and Sobel [9] (hereinafter referred to as CS) is the most successful one describing cheap talk with private information. We employ the CS model as the basis of the environment that we consider in the present paper and extend it to the situation in which an exit option is available.

The CS model has two players. One player possesses private information about the current state of the relationship, which is randomly drawn. In order to transmit the information, he sends a costless message to his partner, and the latter responds with a decision affecting both the players' payoffs. In CS, the latter is called the Receiver (R) and the former is called the Sender (S). CS shows that the incongruence between both players' preferences restricts the informativeness of cheap talk; in particular, they demonstrate that perfect information transmission via cheap talk is impossible as an equilibrium behavior unless the players' preferences completely coincide.

In the present paper, we assume that S has an exit option after he observes R's decision. When S exercises the exit option, both players obtain their exit payoffs independent of the action chosen by R. The key feature of our results is the difference between the players' payoff when S chooses to stay and an optimal action is chosen (maximum stay payoff) and one when she chooses to exit (exit payoff). Consider the case where R's difference is large and S's difference is small but positive. In this case, R has a strong incentive to prevent S from choosing the exit option, and therefore, R will make a decision that is desirable for S even if both players' preferences differ. Expecting R's response, S has a strong incentive to transmit more accurate information via cheap talk. It follows that the existence of S's exit option increases the informativeness of cheap talk, which in turn may increase not only S's payoff but also that of R. Moreover, we show that as S's difference approaches 0, the information transmission via cheap talk in the most informative equilibrium becomes almost perfect. In other words, the exit reinforces the voice in that the existence of the exit increases the informativeness of the voice. This is the main finding of this paper.

CS also shows that on the most efficient equilibrium, the more congruent both players' preferences are, the more informative is the cheap talk. In other words, when the exit option is not available, the informativeness of cheap talk is determined mainly by the degree of incongruence between the players' preferences. However, when the exit option is available, another determinant of the informativeness comes into play: *the credibility of the exit.* A

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message.

smaller difference between S's maximum stay payoff and exit payoff makes his choice of the exit option more credible, which in turn enables a more informative cheap talk transmission on the equilibrium. Thus, we show that informative cheap talk transmission can be carried out even if S's preference is not exactly similar to that of R. Our main claim—the exit reinforces the voice in that the credibility of the exit increases the informativeness of the voice—is consistent with Hirschman's [15].

To the author's knowledge, the exit-voice perspective has seldom been analyzed in any formal model in economics despite the vast citations.[4] Banerjee and Somanathan's study [2] is one exception in that it presents a game-theoretical model of voice. Like us, they consider the voice as an activity of sending a cheap talk message. However, their model differs from ours in some respects. First, they do not consider the exit option, and therefore, they do not investigate the interplay between the exit and voice, which the present paper focuses on. However, they consider the collective aspect of voice formation, which is abstracted out from our model. In this regard, the present paper can be considered as a complement to their paper. Gehlbach [13] presents a formal model of exit and voice. In his model, the voice is considered as some costly activity of gathering the members' various opinions, consolidating them, and bargaining with the leader of the organization. However, in his model, there is no asymmetric information, and therefore, the voice does not play the role of an information transmitter. Although his model sheds the light on one aspect of voice, in this paper, we mainly analyze the role of voice as an information transmitter.

Apart from the exit-voice perspective, the CS model *per se* has attracted considerable attention and has been extended to various directions.[5] However, the exit option's effect on cheap talk has rarely been analyzed. An exception, Matthews [23], deals with a cheap talk game with a congress and a president—the receiver and sender, respectively—with veto power, which is similar to the exit option in our model. In particular, the timing of the events in his model is approximately the same as that in ours. However, there is a large difference with respect to what private information pertains to. In Matthews, private information concerns the sender's preference, while in our model, it pertains to the current state of the relationship. One may consider such a difference to be small, but it leads to very different outcomes: in Matthews, the informativeness of cheap talk is constrained with

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[4] For efforts in the field of political science, see, for example, the survey by Dowding et al. [11].

[5] For example, see Krishna and Morgan [19], Battaglini [4], and Chen et al. [7], among others.

a strict upper bound, independent of the exit payoff. However, in our model, we show that an equilibrium can be close to that with perfect information transmission to any degree. In other words, Matthews does not emphasize that the existence of the exit increases the informativeness of cheap talk, which is the main claim of the present paper.

Like CS, we do not allow the players to design a mechanism or contract dependent upon the message sent by S. However, in the literature on delegation, such as Holmström [16] and Melumad and Shibano [24], it is often assumed that R can commit to message-dependent mechanisms.[6] In this context, our result implies that when an exit option is available, an efficient outcome can be realized through a simple contract that allocates the joint surplus so that S's difference is small and R's difference is large even if R *cannot* commit to the message-dependent mechanisms.

Dessein [10] and Marino [21] compare the outcome of simple delegation without any message-dependent mechanism with that of cheap talk. The results of the present paper are also related to them. In the present paper, we assume that R cannot delegate her choice of action to S. Despite this assumption, our result implies that there is a sequence of equilibrium actions converging to the most desirable action for S. This outcome is realized if R commits to delegating the choice of the action to S. In other words, our result implies that even if a commitment to delegation is impossible, the credibility of the exit option can bring about a similar outcome.[7]

In addition, Compte and Jehiel [8] and Bester and Krähmer [5] analyze a mechanism design problem when an exit option is available. Since the environments described in their paper are somehow different from ours, the logic of our model does not appear in their models. Compte and Jehiel show that the existence of the exit option makes it more difficult to implement an efficient outcome. This is contrary to the conclusion of our paper: the existence of the exit option leads to more efficient outcome via more informative cheap talk.

The rest of the paper is organized as follows. In Section 2, we present a formal model of exit and voice. In Section 3, we analyze a specific model (called uniform-quadratic environment with constant difference) and present the main claim of this paper. In Section

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[6] For more recent literature, see Baron [3], Krähmer [18], Martimort and Semenov [22], Alonso and Matouschek [1], Mylovanov [25], and Kovac and Mylovanov [17].

[7] Bester and Strausz [6], [2], and Krishna and Morgan [20] also analyze a mechanism design problem when only partial commitment of delegation is possible.

4, we present a sufficient condition for the main claim to hold in a general model. Finally, in Section 5, we discuss and summerize the results.

2 Setup

There are two players, namely, the sender (S) and the receiver (R). At the beginning of the game, the current state of the relationship between S and R, $t \in T$ is randomly chosen according to a probability distribution $F(t)$. A realized state is observed by S but not by R. On the basis of this observation, S chooses a message $m \in M$ to be sent to R. This message is cheap talk in that it is payoff-irrelevant. After R receives S's message, R chooses an action $a \in A$ relevant to both players' payoffs.

Up to this point, all elements are the same as in CS. Now, we introduce the concept of exit. After observing R's action, S chooses whether to exit or stay. If S chooses to exit, S's and R's payoffs are $U^S(t)$ and $U^R(t)$, respectively. If S chooses to stay, S's and R's payoffs are given by $y^S(t,a)$ and $y^R(t,a)$, respectively.

We assume that $T = M = [0,1]$ and $A = \mathbb{R}$. $F(t)$ has a continuous density $f(t)$, where $f(t) > 0$ for any $t \in T$. We assume that for $i = S, R$, $\partial y^i(t,a)/\partial a$ is defined and continuously differentiable, and

$$\begin{aligned}
&\forall t\ \exists a \text{ such that } \frac{\partial y^i(t,a)}{\partial a} = 0,\\
&\forall t, \forall a,\ \frac{\partial^2 y^i(t,a)}{\partial a^2} < 0,\\
&\forall t, \forall a,\ \frac{\partial^2 y^i(t,a)}{\partial a \partial t} > 0.
\end{aligned}$$

Furthermore, we assume that $U^i(t)$ is continuously differentiable in t for $i = S, R$.

Remark 1 Our model assumes that only S has the exit option. Even if R also has the exit option, our results would not change, provided S keeps the exit option, since in our model, R can virtually induce S to choose the exit option by choosing some extreme action in our model. However, it is easily observed that there would be a dramatic change if S no longer has an exit option. This consideration suggests that whether or not the player with the voice option has the exit option is a relevant factor.

We consider a perfect Bayesian equilibrium as an equilibrium concept. We also focus only on the class of equilibria with pure strategies. A pure strategy perfect Bayesian equilibrium is defined by $(\mu, p, \alpha, \epsilon)$, where

- $\mu : T \to M$: S's message strategy,
- $p : M \times T \to [0,1]$: R's posterior belief density function over T on the observation of m,
- $\alpha : M \to A$: R's action choice strategy, and
- $\epsilon : T \times A \to \{0,1\}$: S's exit strategy. To be more precise, $\epsilon = 1$ refers to exit and $\epsilon = 0$ refers to stay.

The equilibrium conditions are

$$\mu(t) \in \arg\max_{m \in M} \left\{\epsilon(t,\alpha(m))U^S + (1-\epsilon(t,\alpha(m)))y^S(t,\alpha(m))\right\}, \quad \forall t \in T,$$

$$\int_{t' \in T} f(t')\mathbb{I}\{\mu(t') = m\}dt' > 0 \Rightarrow p(m,t) = \frac{f(t)\mathbb{I}\{\mu(t) = m\}}{\int_{t' \in T} f(t')\mathbb{I}\{\mu(t') = m\}dt'} \quad (\mathbb{I}\text{: identity function}),$$

$$\alpha(m) \in \arg\max_{a \in A} \int_{t \in T} \left\{\epsilon(t,a)U^R + (1-\epsilon(t,a))y^R(t,a)\right\} p(m,t)dt, \quad \forall m \in M,$$

$$\forall t \in T, \forall a \in A, \begin{cases} U^S > y^S(t,a) & \Rightarrow \epsilon(t,a) = 1, \\ U^S < y^S(t,a) & \Rightarrow \epsilon(t,a) = 0. \end{cases}$$

The first line refers to the condition that $\mu(t)$ is an optimal message for type t of S, given R's strategy and S's exit strategy. The second line refers to the condition that R's posterior belief is updated by adhering as much as possible to the Bayesian approach. The third line refers to the condition that $\alpha(m)$ is an optimal action for R, given R's posterior belief and S's exit strategy. The last line refers to the condition that $\epsilon(t,a)$ is an optimal exit choice for type t of S, given a realized action a.

We state that an action a is *induced on the equilibrium path* if there exists $t \in T$ such that $a = \alpha \circ \mu(t)$ and $\epsilon(t,a) = 0$. For ease of exposition, given an interval $\tau \subseteq T$ where $\inf \tau = \underline{t}$ and $\sup \tau = \bar{t}$, the posterior density function based on the observation that $t \in \tau$ is denote by f_τ; that is

$$f_\tau(t) = \begin{cases} \frac{f(t)}{F(\bar{t}) - F(\underline{t})} & \text{if } t \in \tau, \\ 0 & \text{otherwise.} \end{cases}$$

3 Uniform-Quadratic Environment with Constant Difference

In this section, we employ a more specific model, which we call *a uniform-quadratic environment with constant difference.* In this model, $F(t)$ is a uniform distribution function on $[0,1]$ and S's and R's stay payoffs are respectively expressed as

$$\begin{aligned} y^S(t,a) &= Y^S - (t+b-a)^2, \\ y^R(t,a) &= Y^R - (t-a)^2, \end{aligned}$$

for some $b > 0$. Here, b is called *a bias* that represents a degree of incongruence between S's and R's optimal actions.[8] Y^i is the maximum stay payoff for $i = S, R$. A uniform-quadratic environment was originally analyzed in Section 4 of CS. Furthermore, we assume that both players' exit payoffs are independent of t. We define the difference between i's maximum stay payoff and exit payoff by $D^i = Y^i - U^i$ for $i = S, R$. Throughout this section, we assume that $D^S > 0$ and $D^R > 0$.

3.1 Preliminary Results: Environment without an Exit Option

We first revisit CS's results in an environment without an exit option. If the exit option is not available, perfect information transmission via cheap talk does not occur. This is because S has no incentive to truthfully report the current state for fear of R exploiting the information.

To be more precise, CS shows that in any equilibrium, there are finite intervals partitioning T and S informs R about which interval a true state is lying in via cheap talk. The necessary and sufficient condition for the existence of the equilibrium with N intervals is

$$b < \left\langle \frac{1}{2N(N-1)} \right\rangle, \tag{1}$$

where $\langle \cdot \rangle$ is the operator such that

$$\left\langle \frac{x}{y} \right\rangle = \begin{cases} \frac{x}{y}, & \text{if } y \neq 0, \\ \infty, & \text{if } y = 0,\ x \neq 0. \end{cases}$$

In other words, regarding N as the informativeness of the cheap talk, the informativeness is determined by the bias b. The smaller b is, the more intervals the equilibrium has.

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[8] For interpretations of biases in the real world, see the discussion in Dessein [10], among others.

Henceforth, we only focus on the most informative equilibrium or the equilibrium with the most intervals.[9] Indeed, CS shows that the equilibrium with the most intervals is Pareto superior to any other equilibrium with fewer intervals.

3.2 Characterization of No-Exit Equilibria

Hereafter, we consider the environment in which an exit option is available for S. To see what is brought about by the introduction of the exit option, consider the case in which $Y^S = 1$, $b = \frac{\sqrt{10}}{12}$, and D^R is so large that R has a strong incentive to avoid S's exit. If the exit option is not available, (1) implies that there exists only a babbling equilibrium. Even if the exit option is available, when S's exit payoff is relatively small (precisely, $U^S \leq \left(\frac{3}{2} + b\right)\left(\frac{1}{2} - b\right)$), the existence of the exit option has no effect on equilibrium behavior, and therefore, the babbling equilibrium is still the unique equilibrium.

What will happen when S's exit payoff rise? Figure 1 illustrates the situation with $U^S = \left(\frac{3}{2} + b\right)\left(\frac{1}{2} - b\right)$ where the horizontal axis, the vertical axis, the solid curve, and the dotted line express the state t, S's payoff, S's stay payoff, and S's exit payoff, respectively. In this situation, S is indifferent between stay and exit at state 1. Then, when S's exit payoff rises slightly, S will choose the exit option around state 1. To avoid this, R chooses an action more desirable for S. Such an effort by R is possible up to the situation illustrated in Figure 2 (precisely, $U^S = \frac{3}{4}$). In this situation, since S is already indifferent between stay and exit at both ends of the state space, it may seem impossible to avoid S's exit at every state when S's exit payoff goes up further. However, in such a situation, an equilibrium with two intervals, where the exit option is not chosen on the equilibrium path, appears. No-exit equilibria with two intervals can be sustained up to the situation illustrated in Figure 3 (precisely, $U^S = \frac{15}{16}$). Similarly, when S's exit payoff rises further (up to $U^S = \frac{35}{36}$), there are no-exit equilibria with 3 intervals, as illustrated in Figure 4. Below, we will see that as S's exit payoff approaches her maximum stay payoff, the equilibria have more intervals.

<Figures 1-4 should be inserted>

Hereafter, we focus on the equilibrium in which an exit option is never exercised on the equilibrium path. We call it *no-exit equilibrium (NEE)*. The following result asserts that

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[9] Che et al. [7] present a condition that selects the most informative equilibrium in uniform-quadratic models.

any NEE is characterized by a partition of the state space consisting of finite intervals.[10]

Lemma 1 In any equilibrium, there are only finite actions induced on the equilibrium path. This implies that any NEE $(\mu, p, \alpha, \epsilon)$ is characterized by a partition $\{\tau_n\}_{n=1,\dots,N}$ of $[0,1]$ such that

- N is finite;
- τ_n is an interval for $n = 1, \dots, N$; and
- there exist $\{t_n\}_{n=0,\dots,N}$, $\{m_n\}_{n=1,\dots,N}$, $\{a_n\}_{n=1,\dots,N}$ such that
  - $\inf \tau_n = t_{n-1}$ and $\sup \tau_n = t_n$ for $n = 1, \dots, N$,
  - $0 = t_0 < t_1 < \cdots < t_N = 1$,
  - $\mu(t) = m_n$ for any $t \in \tau_n$, $m_n \neq m_{n'}$ for $n \neq n'$, and therefore, $p(m_n, t) = f_{\tau_n}(t)$ for $n = 1, \dots n$,
  - $\alpha(m_n) = a_n$ for $n = 1, \dots, N$, and
  - $\epsilon(t, a_n) = 0$ for $t \in \tau_n$ and $n = 1, \dots, N$.

All proofs are relegated to the the Appendix. Below, we derive the equilibrium condition for NEE with N intervals. The following result shows that any interval of NEE is classified into three categories.

Lemma 2 Fix an NEE and an interval $\hat{\tau}$. Let $\inf \hat{\tau} = \underline{t}$, $\sup \hat{\tau} = \bar{t}$, and $\hat{a} = \alpha \circ \mu(t)$ for $t \in \hat{\tau}$. Then, $\bar{t} - \underline{t} \leq 2\sqrt{D^S}$. Moreover, $\hat{\tau}$ belongs to either one of the following categories:

Interval $\mathcal{N}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

- $\bar{t} - \underline{t} < 2\sqrt{D^S} - 2b$,
- $\hat{a} = \frac{\underline{t}+\bar{t}}{2}$,
- $y^S(\underline{t}, \hat{a}) > U^S$, and
- $y^S(\bar{t}, \hat{a}) > U^S$.

Interval $\mathcal{A}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

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[10] Note that an equilibrium may have infinite number of intervals in our model. This is because there are infinite actions inducing S to choose the exit option.

- $2\sqrt{D^S} > \bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b$,
- $\hat{a} = \bar{t} - \sqrt{D^S} + b$,
- $y^S(\underline{t}, \hat{a}) > U^S$, and
- $y^S(\bar{t}, \hat{a}) = U^S$.

Interval $\mathcal{F}$: $\underline{t}$, $\bar{t}$, and $\hat{a}$ satisfy

- $\bar{t} - \underline{t} = 2\sqrt{D^S}$,
- $\hat{a} = \bar{t} - \sqrt{D^S} + b$, and
- $y^S(\underline{t}, \hat{a}) = y^S(\bar{t}, \hat{a}) = U^S$.

Furthermore, in any interval, the receiver has an incentive to choose $\hat{a}$ if $\sqrt{D^R} \geq \sqrt{D^S} + b$.

Interval $\mathcal{N}$ is a non-accommodating interval in the sense that R can choose her best action without fearing of S's exit. Interval $\mathcal{A}$ is an accommodating interval in that the constraint for no exit is binding at the right end of the interval and R chooses an action more favorable for S than the one most favorable for R. Interval $\mathcal{F}$ is a fully accommodating interval in that the constraint for no exit is binding at both ends of the interval and any other action than $\hat{a}$ induces some types of S to exit. This lemma also asserts that any NEE interval cannot be longer than $2\sqrt{D^S}$, for otherwise some types of S will choose to exit irrespective of the action chosen by R.

The important fact is that on the boundary point of two adjoining intervals, S must be indifferent between sending messages corresponding to the intervals. This implies that possible equilibrium configurations of intervals are restricted. For example, an interval $\mathcal{F}$ cannot be directly connected to an interval $\mathcal{N}$, for otherwise S will have a strict incentive to choose the action corresponding to the interval $\mathcal{N}$ at any state sufficiently close to the boundary point. By exhausting all possibilities, we can derive the equilibrium condition for S. The following is the formal statement:

Lemma 3 Given any NEE with N intervals,

(i) for $N = 1$, the equilibrium condition for the sender is $\sqrt{D^S} \geq \frac{1}{2N}$, and

(ii) for $N \geq 2$, a configuration of intervals is either of the following five patterns:

(I) $\mathcal{N}, \dots, \mathcal{N}$,

(II) $\mathcal{N}, \dots, \mathcal{N}, \mathcal{A}$,

(III) $\mathcal{N}, \dots, \mathcal{N}, \mathcal{A}, \mathcal{F}, \dots, \mathcal{F}$,[11]

(IV) $\mathcal{A}, \mathcal{F}, \dots, \mathcal{F}$, or

(V) $\mathcal{F}, \dots, \mathcal{F}$.

The equilibrium condition for the sender in each case is the following:[12]

(I) $b < \left\langle \frac{1}{2N(N-1)} \right\rangle$ and $\sqrt{D^S} > \frac{1}{2N} + Nb$.

(II) $\frac{1}{2N} + \frac{(N-1)^2}{N} b < \sqrt{D^S} \le \frac{1}{2N} + Nb$ and $\sqrt{D^S} < 1 - (2N^2 - 4N + 1)b$.

(III) $\frac{1}{2N} + \frac{(i-1)^2}{N} b < \sqrt{D^S} \le \frac{1}{2N} + \frac{i^2}{N} b$ and $\sqrt{D^S} < \frac{1-(2i^2-4i+1)b}{2N-2i+1}$ for some $i = 2, \dots, N-1$.

(IV) $\frac{1}{2N} < \sqrt{D^S} \le \frac{1}{2N} + \frac{1}{N} b$ and $\sqrt{D^S} < \left\langle \frac{1}{2(N-1)} \right\rangle$.

(V) $\sqrt{D^S} = \frac{1}{2N}$.

These conditions are illustrated in Figure 5. Combined with these lemmas, we derive the equilibrium condition for NEE.

<Figures 5 should be inserted>

Theorem 1 Suppose $\sqrt{D^R} \ge \sqrt{D^S} + b$. Then, an NEE with N intervals exists if and only if both (1) and (2) hold:

(1) Either one of (1-1)–(1-3) holds:

  (1-1) $b < \left\langle \frac{1}{2N(N-1)} \right\rangle$,

  (1-2) $\sqrt{D^S} < \frac{1-(2i^2-4i+1)b}{2N-2i+1}$ for some $i = 2, \dots, N$, or

  (1-3) $\sqrt{D^S} < \left\langle \frac{1}{2(N-1)} \right\rangle$.

(2) $\sqrt{D^S} \ge \frac{1}{2N}$.

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[11] This case occurs only if $N \ge 3$.

[12] For the definition of the operator $\langle \cdot \rangle$, see Section 3.1.

This theorem asserts that an NEE with sufficiently large number of intervals exists if and only if b is sufficiently small and/or D^S is sufficiently small as long as D^R is sufficiently large. Identifying the equilibrium number of intervals with *the informativeness of cheap talk*, we can interpret this result as follows: in our model, there are two determinants of the informativeness of cheap talk. One is the smallness of b, which refers to the degree of incongruence between S's and R's preferences. This is extensively discussed by CS and in other literature. The other is the smallness of D^S, which is newly found. We interpret the smallness of D^S as a degree of *S's credibility of exit*. In other words, the smaller D^S is, the more credible S's threat of exit is and the more informative information cheap talk can convey. Henceforth, we call such an NEE *an NEE driven by the credibility of exit.*[13]

Furthermore, the previous theorem implies the following important facts:

Corollary 1 Suppose that $\sqrt{D^R} > b$. Then, as U^S approaches Y^S (equivalently, D^S approaches 0), there exists a sequence of equilibria in which $\alpha \circ \mu(t)$ converges pointwise to $t + b$.

This corollary implies that approximately perfect information transmission is possible via cheap talk in the limit. The corollary also implies that the equilibrium actions converge to the optimal action for R at any state. In other words, our result implies that even if a commitment to delegation is impossible, the credibility of the exit option can bring about a similar outcome.

Corollary 2 Suppose that $\sqrt{D^R} > b$ and $b < \frac{1}{2\sqrt{3}}$. Then, if U^S is sufficiently close to Y^S, there exists an equilibrium in the environment with the exit, in which S's and R's ex ante payoffs are both larger than those in any equilibrium in the environment without the exit.

This corollary implies that the existence of S's exit option increases the ex ante payoff of R as well as S. Therefore, giving S an exit option is Pareto-improving.

Remark 2 CS model is often used in the literature on mechanism design, such as in Holmström [16] and Melumad and Shibano [24]. Basically, in these studies, it is assumed that R can commit to the mechanism depending upon the message sent by S. In this context, our result implies that in the environment with an exit option, an efficient outcome

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[13] To be more precise, an NEE driven by the credibility of exit is an NEE with a configuration of intervals (IV) or (V) in Lemma 3.

can be realized by a simple contract even if R *cannot* commit to the message-dependent mechanism.

To observe this, let us consider the situation in which players' maximum stay payoffs are determined by the splitting of the joint surplus Y. Before a state is realized, R proposes a contract that specifies an allocation of Y between S's share Y^S and R's share Y^R. This contract does not depend upon the message. If $Y > U^S + U^R + b^2$, an allocation with $Y^S = U^S + \varepsilon$ and $Y^R = Y - Y^S$ for sufficiently small ε makes an almost perfect information transmission possible because it satisfies the premise of Theorem 1 for a large N.

4 General Model

In this section, we extend the results obtained in the uniform-quadratic environment with constant difference to more broad environments. More precisely, we derive a sufficient condition for the existence of NEE driven by the credibility of exit in the general setting.

On the basis of the single-peakedness, we can identify a unique maximizer of $y^i(t, \cdot)$ for any t. It is denoted by $\sigma^i(t)$. By the assumptions on y^i, it is verified that $\sigma^i(t)$ is continuously differentiable and strictly increasing in t. We posit the following assumption:

Assumption 1 Either one of the following conditions hold:

(a) $\sigma^S(0) > \sigma^R(0)$.

(b) $\sigma^R(1) > \sigma^S(1)$.

When (a) holds, define $b = \sigma^S(0) - \sigma^R(0)$, and when (b) holds, $b = \sigma^R(1) - \sigma^S(1)$.

Roughly speaking, Assumption 1 requires that players' optimal actions differ in either end of the state space in some direction. This assumption holds in the models in which players' stay payoffs have a form of quadratic loss function with constant bias.

Let $Y^i(t) = y^i(t, \sigma^i(t))$ be i's maximum stay payoff at state t. Let us denote the difference between i's maximum stay payoff and exit payoff at state t by $D^i(t) = Y^i(t) - U^i(t)$. Whenever $D^S(t) > 0$, we can uniquely define $\gamma_-(t)$ and $\gamma_+(t)$ such that

$$\begin{aligned}
&\gamma_-(t) < \gamma_+(t), \\
&y^S(t, \gamma_-(t)) = y^S(t, \gamma_+(t)) = U^S(t).
\end{aligned}$$

In other words, $\gamma_+(t)$ and $\gamma_+(t)$ are the actions that makes S indifferent between stay and exit at state t. On the basis of the assumptions on y^S and U^S, it is verified that γ_+ and γ_- are continuously differentiable and Lipschitz continuous in t, and

$$\gamma_-(t) < \sigma^S(t) < \gamma_+(t)$$

holds for any t. We posit the following assumption:

Assumption 2 Under Assumption 1,

- if (a) is required in Assumption 1, then $\gamma_-(t)$ must be strictly increasing in t, or
- if (b) is required in Assumption 1, then $\gamma_+(t)$ must be strictly increasing in t.

This assumption is met in the uniform-quadratic environment with constant difference.

We can show that under these assumptions, if $\gamma_+(t) - \gamma_-(t)$ is sufficiently small, there exists an NEE with many intervals.

Theorem 2 Under Assumptions 1 and 2, suppose that $D^R(t)$ is sufficiently large and $D^S(t) > 0$ for any t. Then, for any natural number $N > \underline{N}$, there exists an NEE with N or more intervals if the condition $\gamma_+(t) - \gamma_-(t) \leq \overline{\gamma}$ holds for any t. Here,[14]

$$\begin{aligned}
\underline{N} &= \frac{\underline{\delta} + 2\overline{\delta}}{2b}, \\
\overline{\gamma} &= \frac{\underline{\delta}}{2(N-1)}, \\
\underline{\delta} &= \inf_{t>t'} \frac{\sigma^S(t) - \sigma^S(t')}{t - t'}, \\
\overline{\delta} &= \sup_{t>t'} \frac{\sigma^R(t) - \sigma^R(t')}{t - t'}.
\end{aligned}$$

Moreover, the length of each interval can be made less than $\frac{1}{N-1}$.

Note that the theorem does not depend upon the specifications of distribution functions on t.

If S's difference $D^S(t)$ is independent of t, we can reduce $\gamma_+(t) - \gamma_-(t)$ to any degree by letting D^S approach to 0; this implies that perfect information transmission via cheap talk can be approximated.

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[14] It is verified that $\underline{\delta} > 0$ and $\overline{\delta} < \infty$.

Corollary 3 Under Assumption 1, suppose that $D^R(t)$ is sufficiently large for any t. If $D^S = D^S(t)$ for any t, then there exists a sequence of equilibria in which $\alpha \circ \mu(t)$ converges pointwise to $\sigma^S(t)$ as D^S approaches 0.

Assumption 1 is much less strict than the constant bias assumption made by the ordinary uniform-quadratic model. Consider the following example:

Example 1 [15] We assume that both players' exit payoffs are constant, and

$$\begin{aligned} y^S &= Y^S - (t-a)^2, \\ y^R &= Y^R - (ct-b-a)^2, \end{aligned}$$

where $b > 0$ and $c > 0$. Furthermore, $F(t)$ is uniform. Then, since

$$\begin{aligned} \sigma^S(t) &= t, \\ \sigma^R(t) &= ct - b, \end{aligned}$$

it is verified that Assumption 1 is met. Note that when $c > 1 + b$, we obtain

$$\begin{aligned} \sigma^S(0) &> \sigma^R(0), \\ \sigma^S(1) &< \sigma^R(1). \end{aligned}$$

In other words, the sign of the incongruence between S's and R's preferences is reversed.

We define N such that

$$\frac{1}{2(N-1)} > \sqrt{D^S} \geq \frac{1}{2N}. \tag{2}$$

Then, as U^S approaches Y^S, N increases to infinity.

We define $t_0 = 0$, and

$$\begin{aligned} t_n &= 1 - 2(N-n)\sqrt{D^S}, \quad n = 1, \ldots, N, \\ a_n &= 1 - (2N - 2n + 1)\sqrt{D^S}, \quad n = 1, \ldots, N. \end{aligned}$$

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[15] This case is a variant of that shown in Melumad and Shibano [24].

We construct a candidate for an equilibrium as follows:

$$\begin{aligned}
&\{\tau_n\}_{n=1,\ldots,N} \text{ is a partition of } [0,1],\\
&\inf \tau_n = t_{n-1} \text{ and } \sup \tau_n = t_n, \quad n = 1,\ldots,N,\\
&\mu(t) = m_n, \quad t \in \tau_n,\\
&p(m_n, t) = f_{\tau_n}(t),\\
&\alpha(m_n) = a_n,\\
&\epsilon(t,a) = 0, \quad \text{iff } y^S(t,a) \geq U^S.
\end{aligned}$$

Since $y^S(t, \alpha \circ \mu(t)) \geq U^S$, the exit option is never chosen on the equilibrium path. However, for $n = 2,\ldots,N$, since $y^S(t_{n-1}, a_n) = y^S(t_n, a_n) = U^S$, if R chooses $\tilde{a} \neq a_n$, some types of S belonging to τ_n would choose the exit option. Therefore, R with a sufficiently small U^R has no incentive to deviate from the equilibrium action a_n.

Consider R's incentive after receiving a signal m_1. Since $y^S(t_1, a_1) = U^S$ and $a_1 > \sigma^S(t_1)$, if R chooses $\tilde{a} < a_1$, some types of S close to t_1 would choose the exit option. Therefore, R with sufficiently small U^R has no incentive to choose $\tilde{a} < a_1$. However, R's expected payoff function *when the exit option is never chosen* is single-peaked and the maximum is attained at

$$a^* = c\mathbb{E}[t|t \in \tau_1] - b.$$

Suppose

$$N \geq \frac{c}{b}.$$

Then, by (2),

$$\begin{aligned}
a^* &= c\mathbb{E}[t|t \in \tau_1] - b\\
&< c\left[1 - 2(N-1)\sqrt{D^S}\right] - b\\
&\leq c\left(1 - \frac{N-1}{N}\right) - b\\
&\leq 0\\
&< 1 - (2N-1)\sqrt{D^S}\\
&= a_1.
\end{aligned}$$

This implies that a deviation $\tilde{a} > a_1$ is never beneficial for R with a sufficiently large Y^S. Thus, it is evident that the above candidate indeed constitutes an equilibrium.

Even for the environments where S's difference is not constant, Theorem 2 gives us a sufficient condition for the existence of NEE with many intervals. Consider the following example:

Example 2 We assume that

$$y^S = Y^S(t) - (t + b - a)^2,$$
$$y^R = Y^R(t) - (t - a)^2,$$

where $b > 0$. Note that Assumption 1 is met. We obtain

$$\gamma_+(t) = t + b + \sqrt{D^S(t)},$$
$$\gamma_-(t) = t + b - \sqrt{D^S(t)}.$$

Then, $\gamma_+(t) - \gamma_-(t) < \overline{\gamma}$ holds for any t if and only if

$$\sqrt{D^S(t)} \leq \frac{1}{4(N-1)} \quad \forall t.$$

On the other hand, Assumption 2, that is, $\frac{d\gamma_-(t)}{dt} > 0$, holds if and only if

$$\sqrt{D^S(t)} > \frac{1}{2}\frac{dD^S(t)}{dt} \quad \forall t.$$

If these conditions hold, Theorem 2 guarantees the existence of NEE with N or more intervals whenever $D^R(t)$ is sufficiently large for any t.

Assumption 2 is crucial for the construction of NEE driven by the credibility of exit. Consider the following example:

Example 3 We assume that both players' exit payoffs are constant and

$$y^S(t, a) = d^2 t + \varepsilon - (t + b - a)^2,$$
$$y^R(t, a) = Y^R(t) - (t - a)^2,$$

where $b > 0$, $\frac{1}{N-1} > d > \frac{2}{2N-1}$ for some natural number N, and ε is a sufficiently small positive real number. This model satisfies Assumption 1. However,

$$\gamma_-(t) = t + b - \sqrt{d^2 t + \varepsilon}.$$

Then, Assumption 2 is violated. Therefore, we cannot construct an NEE driven by the credibility of exit. However, there exists the following equilibrium with N intervals characterized by $\{t_n\}_{n=0}^N$ and $\{a_n\}_{n=1}^N$(as long as $Y^R(t)$ is sufficiently large for any t):

$$\begin{aligned}
&t_N = 1,\\
&t_{n-1} = t_n - 2\sqrt{d^2 t_n + \varepsilon} + d^2 \quad n = N, \dots, 2,\\
&t_0 = 0,\\
&a_1 \text{ is some action satisfying } y^S(t, a_1) < 0 \quad \forall t,\\
&a_n = t_n + b - \sqrt{d^2 t_n + \varepsilon} \quad n = 2, \dots, N.
\end{aligned}$$

where ε is sufficiently small such that

$$\begin{aligned}
&t_1 > 0,\\
&2\sqrt{d^2 t_1 + \varepsilon} < d^2.
\end{aligned}$$

It is verified that such ε indeed exists.

On the equilibrium path, S chooses an exit option when $t \in [0, t_1)$. In order to avoid this, $a \in [b - \sqrt{\varepsilon}, b + \sqrt{\varepsilon}]$ must be chosen, but this action induces some types of S belonging to $[t_1, t_2]$ to deviate from the equilibrium strategy.

The smallness of the upper bound on $(\gamma_+ - \gamma_-)$ is also a crucial condition for Theorem 2. Consider the following example:

Example 4 [16] We assume that both players' exit payoffs are constant and

$$\begin{aligned}
y^S &= (1+t)\sqrt{a} - \frac{1}{\sqrt{1+4b}} a,\\
y^R &= (1+t)\sqrt{a} - a,
\end{aligned}$$

where $b < 0$. In this example, Assumptions 1 and 2 are satisfied. However, it is verified that as long as $U^S < \min Y^S(t)$,

$$\gamma_+(1) - \gamma_-(1) > \sqrt{2}(1+4b)$$

holds. Then, the presupposition of Theorem 2 is not satisfied for sufficiently large N.

In this example, if $y^S(\hat{t}, \hat{a}) = U^S$, then for $t < \hat{t}$, $y^S(t, \hat{a}) < U^S$. This implies that S receives a positive payoff in any boundary point, except at $t = 1$. Therefore, there is no NEE driven by the credibility of exit.

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[16] This case is a special case shown in Marino [21].

5 Concluding Remarks

This paper investigates the interplay between exit and voice by analyzing a modified version of Crawford and Sobel's [9] model in which the sender has an exit option after the receiver makes a decision. The key feature of our results is the difference between players' maximum stay payoff and exit payoff. We find that in the case where the receiver's difference is large and the sender's difference is small but positive, the latter's exit is so credible that the former makes a decision that is desirable to the latter so as to prevent him from exercising the exit option; through this, accurate information can be transmitted via cheap talk on the equilibrium. In other words, it is shown that the informativeness of cheap talk is determined by not only the degree of incongruence between both players' preferences but also the credibility of the sender's exit, which is measured by the smallness of the sender's difference. Furthermore, the perfect information transmission via cheap talk can be approximated in the limit. To the author's knowledge, these results are unprecedented in the literature on cheap talk with private information.

In this paper, we mainly focus on the class of NEE. A final remark should be made with regard to the case in which NEE does not exist. If the receiver's exit payoff is relatively large, the receiver will induce the sender to choose an exit option whenever it is expected that only a rough information cannot be transmitted. Even if the receiver has a strong incentive to prevent the sender from the exit option, she may not be able to do so. Example 3 is such a situation. Nevertheless, according to the equilibrium we have constructed, the sender transmits information to the receiver and chooses to stay at some states while he chooses an exit option at other states. In particular, when d is sufficiently small, quite accurate information is transmitted via cheap talk on the states where the sender chooses to stay. Therefore, the credibility of exit is the source of the informativeness of cheap talk as in NEE.

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Appendix

A Proofs

A.1 Proof of Lemma 1

Suppose, to the contrary, that there exists an equilibrium with infinite actions induced on the equilibrium path. Then, there must exist actions a_1, a_2, and a_3 induced on the equilibrium path such that $a_1 < a_2 < a_3$ and $a_3 - a_1 < \min\{\sqrt{D^R}, b\}$. Taking S's incentive into consideration, the following must hold:

$$\{t|\mu(t) = a_2\} \subseteq (\max\{a_1 - b, 0\}, a_3 - b) \neq \emptyset.$$

However, it can be easily verified that for any $t \in (\max\{a_1 - b, 0\}, a_3 - b)$,

$$\begin{aligned} Y^S - (t + b - a_1)^2 &> U^S, \\ Y^S - (t + b - a_2)^2 &> U^S. \end{aligned}$$

It follows that no type of S would send a message inducing a_2 on the equilibrium and would choose the exit option if R chooses a_1 or a_2. Since

$$-\int_{\{t|\mu(t)=a_2\}} (t - a_1)^2 dt > -\int_{\{t|\mu(t)=a_2\}} (t - a_2)^2 dt,$$

R would deviate to choosing a_1 after receiving a message inducing a_2. This is a contradiction. The latter part of the statement is easily verified on the basis of the assumptions on y^S. ∎

A.2 Proof of Lemma 2

First of all, if $t < a - b - \sqrt{D^S}$ or $t > a - b + \sqrt{D^S}$, then $\epsilon(t, a) = 1$. This implies that the length of any NEE interval must be less than or equal to $2\sqrt{D^S}$. Henceforth, we focus on the case in which $\bar{t} - \underline{t} \leq 2\sqrt{D^S}$.

Let us denote R's expected equilibrium payoff on choosing a conditional on the assumption that $t \in \hat{\tau}$ by $\tilde{V}^R(a)$. Then, we obtain the following expression:

$$W(a) = (\bar{t} - \underline{t})(\tilde{V}^R(a) - U^R) =$$
$$\begin{cases} 0 & \text{if } a \in A_1 = (-\infty, \underline{t} + b - \sqrt{D^S}], \\ (a - b + \sqrt{D^S} - \underline{t})D^R - \int_{\underline{t}}^{a-b+\sqrt{D^S}} (t-a)^2 dt & \text{if } a \in A_2 = (\underline{t} + b - \sqrt{D^S}, \bar{t} + b - \sqrt{D^S}), \\ (\bar{t} - \underline{t})D^R - \int_{\underline{t}}^{\bar{t}} (t-a)^2 dt & \text{if } a \in A_3 = [\bar{t} + b - \sqrt{D^S}), \underline{t} + b + \sqrt{D^S}], \\ (\bar{t} - a + b + \sqrt{D^S})D^R - \int_{a-b-\sqrt{D^S}}^{\bar{t}} (t-a)^2 dt & \text{if } a \in A_4 = (\underline{t} + b + \sqrt{D^S}, \bar{t} + b + \sqrt{D^S}), \\ 0 & \text{if } a \in A_5 = [\bar{t} + b + \sqrt{D^S}, \infty). \end{cases}$$

In any NEE, the optimal action must lie on A_3, for otherwise some type of S would choose an exit option. The unique local maximizer on A_3, denoted by a^*, is

$$a^* = \begin{cases} \frac{\underline{t} + \bar{t}}{2} & \text{if } \bar{t} - \underline{t} < 2\sqrt{D^S} - 2b, \\ \bar{t} + b - \sqrt{D^S} & \text{if } \bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b. \end{cases}$$

Note that $a^* = \hat{a}$ in any case. When $\bar{t} - \underline{t} < 2\sqrt{D^S} - 2b$, it is easily verified that $y^S(\underline{t}, a^*) > U^S$ and $y^S(\bar{t}, a^*) > U^S$ hold. In the case of $\bar{t} - \underline{t} \geq 2\sqrt{D^S} - 2b$ it is easily verified $y^S(\bar{t}, a^*) = U^S$ and

$$y^S(\underline{t}, a^*) \begin{cases} > U^S & \text{if } \bar{t} - \underline{t} < 2\sqrt{D^S}, \\ = U^S & \text{if } \bar{t} - \underline{t} = 2\sqrt{D^S}, \end{cases}$$

hold.

In both cases, it is verified that $\sqrt{D^R} \geq \sqrt{D^S} + b$ implies that $W(a^*) \geq 0$ and W has no local maximum on A_2 and A_4. It follows that a^* is a global optimal action. ∎

A.3 Proof of Lemma 3

Condition (i) is immediately proved from Lemma 2. Throughout the proof, we consider $N \geq 2$. On the boundary point of two adjoining intervals, S must be indifferent between sending actions corresponding to the intervals. By Lemma 2, they must be either of the following cases:

- $\mathcal{N}$-$\mathcal{N}$,
- $\mathcal{N}$-$\mathcal{A}$,
- $\mathcal{A}$-$\mathcal{F}$, or

- $\mathcal{F}$-$\mathcal{F}$.

This implies that possible configurations of intervals of NEE is restricted to (I)–(V).

Consider (I). In this type of equilibrium, by the analysis in CS (see Section 3.1),

$$t_n = nt_1 + 2n(n-1)b, \quad n = 0, \ldots, N,$$

where

$$t_1 = \frac{1 - 2N(N-1)b}{N}.$$

The equilibrium condition is

$$\begin{aligned} & t_1 - t_0 > 0, \\ & t_N - t_{N-1} < 2\sqrt{D^S} - 2b. \end{aligned}$$

Then, we obtain

$$\begin{aligned} b &< \left\langle \frac{1}{2N(N-1)} \right\rangle, \\ \sqrt{D^S} &> \frac{1}{2N} + Nb. \end{aligned}$$

Consider (II). In this type of equilibrium,

$$\begin{aligned} t_n &= \begin{cases} nt_1 + 2n(n-1)b, & n = 0, \ldots, N-1, \\ 1, & n = N, \end{cases} \\ a_n &= \begin{cases} \frac{t_{n-1}+t_n}{2}, & n = 1, \ldots, N-1, \\ 1 - \sqrt{D^S} + b, & n = N. \end{cases} \end{aligned}$$

The equilibrium condition is

$$\begin{aligned} & y^S(t_{N-1}, a_{N-1}) = y^S(t_{N-1}, a_N), \\ & 2\sqrt{D^S} > t_N - t_{N-1} \geq 2\sqrt{D^S} - 2b, \\ & t_1 - t_0 > 0, \\ & t_N - t_{N-1} > 0. \end{aligned}$$

Then, we obtain

$$\begin{aligned} & \frac{1}{2N} + \frac{(N-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + Nb, \\ & \sqrt{D^S} < 1 - (2N^2 - 4N + 1)b, \end{aligned}$$

where

$$t_1 = \frac{2 - 2\sqrt{D^S} - 2(2N^2 - 4N + 1)b}{2N - 1}.$$

Consider (III). Given any $i = 2, \dots, N-1$, consider the following configuration:

$$\underbrace{\mathcal{N}, \dots, \mathcal{N}}_{i-1 \text{ times}}, \mathcal{A}, \underbrace{\mathcal{F}, \dots, \mathcal{F}}_{N-i \text{ times}}.$$

In this type of equilibrium,

$$t_n = \begin{cases} nt_1 + 2n(n-1)b, & n = 0, \dots, i-1, \\ 1 - 2(N-n)\sqrt{D^S}, & n = i, \dots, N, \end{cases}$$

$$a_n = \begin{cases} \frac{t_{n-1}+t_n}{2}, & n = 1, \dots, i-1, \\ t_n - \sqrt{D^S} + b, & n = i, \dots, N. \end{cases}$$

The equilibrium condition is

$$\begin{aligned} & y^S(t_{i-1}, a_{i-1}) = y^S(t_{i-1}, a_i), \\ & 2\sqrt{D^S} > t_i - t_{i-1} \geq 2\sqrt{D^S} - 2b, \\ & t_1 - t_0 > 0, \\ & t_i - t_{i-1} > 0. \end{aligned}$$

Then, we obtain

$$\begin{aligned} & \frac{1}{2N} + \frac{(i-1)^2}{N} b < \sqrt{D^S} \leq \frac{1}{2N} + \frac{i^2}{N} b, \\ & \sqrt{D^S} < \frac{1 - (2i^2 - 4i + 1)b}{2N - 2i + 1}, \end{aligned}$$

where

$$t_1 = \frac{2 - 2(2N - 2i + 1)\sqrt{D^S} - 2(2i^2 - 4i + 1)b}{2i - 1}.$$

Consider (IV). In this type of equilibrium,

$$t_n = \begin{cases} 0, & n = 0, \\ 1 - 2(N-n)\sqrt{D^S}, & n = 1, \dots, N. \end{cases}$$

The equilibrium condition is

$$\begin{aligned}&2\sqrt{D^S} > t_1 - t_0 \geq 2\sqrt{D^S} - 2b,\\&t_1 - t_0 > 0.\end{aligned}$$

Then, we obtain

$$\begin{aligned}&\frac{1}{2N} < \sqrt{D^S} \leq \frac{1}{2N} + \frac{1}{N}b,\\&\sqrt{D^S} < \frac{1}{2(N-1)}.\end{aligned}$$

The derivation of the equilibrium condition for (V) is immediate. Since in this type of equilibrium,

$$t_n = 2\sqrt{D^S}n, \quad n = 0, \dots, N,$$

it must hold that $t_N = 1$, or equivalently,

$$\sqrt{D^S} = \frac{1}{2N}.$$

$\blacksquare$

A.4 Proof of Corollary 1

It is obtained directly from Theorem 1 and on the basis of the fact that each interval has a length of $2\sqrt{D^S}$ or less (Lemma 2). $\blacksquare$

A.5 Proof of Corollary 2

By Corollary 1, it is obvious that the sequence of S's ex ante equilibrium payoffs V^S converges to Y^S as $D^S \to 0$. Similarly, as for R's ex ante equilibrium payoff V^R,

$$V^R - V^S = Y^R - Y^S + b\int_0^1 (2t + b - 2\alpha \circ \mu(t))^2 dt \to Y^R - Y^S - b^2$$

as $D^S \to 0$. Then, V^R converges to $Y^R - b^2$. On the other hand, according to CS, S's and R's largest equilibrium ex ante payoffs in the environment without the exit are as follows:

$$\begin{aligned}&\hat{V}^S = Y^S - \frac{4N^2(N^2+2)b^2 + 1}{12N^2},\\&\hat{V}^R = Y^R - \frac{4N^2(N^2-1)b^2 + 1}{12N^2},\end{aligned}$$

respectively, where N is the largest natural number satisfying (1). By a direct calculation, if $b < \frac{1}{2\sqrt{3}}$, then

$$
\begin{aligned}
Y^S &> \hat{V}^S, \\
Y^R - b^2 &> \hat{V}^R.
\end{aligned}
$$

This completes the proof. ▮

A.6 Proof of Theorem 2

We suppose that the presupposition of Theorem 2 holds in any lemmas appearing in this proof. Further, in this proof, we suppose that $\sigma^S(0) > \sigma^R(0)$ and $b = \sigma^S(0) - \sigma^R(0)$. When $\sigma^R(1) > \sigma^S(1)$, we can prove the proposition by reversing all the variables in the following proof at the center of point $1/2$.

First, we prove the following lemma:

Lemma 4 Given any $\ell > 0$ and suppose $\gamma_+(t) - \gamma_-(t) \leq \frac{\underline{\delta}\ell}{2}$ for any t. Then, for any $\tilde{t} \geq \ell$, there exists $\hat{t}$ such that $\hat{t} \in (\tilde{t} - \ell, \tilde{t})$, $\gamma_+(\hat{t}) = \gamma_-(\tilde{t})$, and $y^S(t, \gamma_-(\tilde{t})) > U^S(t)$ for any $t \in (\hat{t}, \tilde{t})$.

Proof:
On the basis of the presupposition, $\sigma^S(\tilde{t}) - \sigma^S(\tilde{t} - \ell) \geq \underline{\delta}\ell$, but

$$
\begin{aligned}
\gamma_-(t) &> \sigma^S(t) - \frac{\underline{\delta}\ell}{2}, \\
\gamma_+(t) &< \sigma^S(t) + \frac{\underline{\delta}\ell}{2}
\end{aligned}
$$

hold for any t. Therefore,

$$
\gamma_-(\tilde{t}) - \gamma_+(\tilde{t} - \ell) > \sigma^S(\tilde{t}) - \sigma^S(\tilde{t} - \ell) - \underline{\delta}\ell \geq 0.
$$

Since γ_+ is continuous in t and $\gamma_-(\tilde{t}) < \gamma_+(\tilde{t})$, we can define $\hat{t} = \max\{t | \gamma_+(t) = \gamma_-(\tilde{t})$ such that $\hat{t} \in (\tilde{t} - \ell, \tilde{t})$. Furthermore, for any $t \in (\hat{t}, \tilde{t})$,

$$
\gamma_-(t) < \gamma_-(\tilde{t}) < \gamma_+(t),
$$

where the first inequality is implied by Assumption 2. Then, it follows that $y^S(t, \gamma_-(\tilde{t})) > U^S(t)$. ▮

Next, we prove the following lemma:

Lemma 5 There exists $\varepsilon > 0$ such that $t > t'$ and $\gamma_-(t) = \gamma_+(t')$ imply $t - t' \geq \varepsilon$.

Proof:

The assumption that $D^S(t) > 0$ for any t implies that $\gamma_+(t) - \gamma_-(t) > 0$ for any t. Then, $\min_t \{\gamma_+(t) - \gamma_-(t)\}$ exists and is strictly positive. Therefore,

$$\gamma_-(t) = \gamma_+(t') \geq \gamma_-(t') + \min_t \{\gamma_+(t) - \gamma_-(t)\}$$

holds. However, on the basis of Lipschitz continuity of γ_-, we obtain

$$t - t' \geq \frac{\gamma_-(t) - \gamma_-(t')}{\ell},$$

where $\ell > 0$ is Lipschitz constant. Finally,

$$t - t' \geq \frac{\min_t \{\gamma_+(t) - \gamma_-(t)\}}{\ell}$$

holds, which implies that we obtain the lemma by setting $\varepsilon = \frac{\min_t \{\gamma_+(t) - \gamma_-(t)\}}{\ell}$. ∎

By using these lemmas, we recursively define a sequence $\{\hat{t}_n\}_{n=0}^{\hat{N}}$ in T as follows: first, we define $\hat{t}_0 = 1$. For $n \geq 0$,

1. if $\hat{t}_n = 0$, we stop the recursive process and denote n by $\hat{N}$;
2. if $\hat{t}_n > 0$ and there exists $\hat{t} \in T$ such that $\gamma_+(\hat{t}) = \gamma_-(\hat{t}_n)$, we define $\hat{t}_{n+1} = \hat{t}$; and
3. if $\hat{t}_n > 0$ and there exists no $\hat{t} \in T$ such that $\gamma_+(\hat{t}) = \gamma_-(\hat{t}_n)$, then we define $\hat{t}_n = 0$.

Lemma 4 implies that $\{\hat{t}_n\}_{n=0}^{\hat{N}}$ is a strictly decreasing sequence, and Lemma 5 implies that $\hat{N}$ is necessarily finite and $\hat{t}_{\hat{N}} = 0$. Lemma 4 also implies that if $\gamma_+(t) - \gamma_-(t) \leq \overline{\gamma}$ for any t, then $\hat{t}_{n-1} - \hat{t}_n < \frac{1}{N-1}$ for any n, and therefore, $\hat{N} \geq N$.

On the basis of the construction of $\{\hat{t}_n\}$, the following lemma is easily verified (the proof is omitted):

Lemma 6

$\forall n = 1, \ldots, \hat{N},\ \forall t \in [\hat{t}_n, \hat{t}_{n-1}],\ y^S(t, \gamma_-(\hat{t}_{n-1})) \geq U^S(t)$,
$\forall n = 1, \ldots, \hat{N} - 1,\ \forall \hat{a} \neq \gamma_-(\hat{t}_{n-1}),\ \exists \hat{t} \in (\hat{t}_n, \hat{t}_{n-1})$ such that $y^S(t, \hat{a}) < U^S(t)\ \forall t \in [\hat{t}_n, \hat{t})$ or $\forall t \in (\hat{t}, \hat{t}_{n-1}]$.

Let $V^R(a; \underline{t}, \overline{t})$ be R's expected payoff of choosing a conditional on the information that $t \in [\underline{t}, \overline{t}]$. Let us denote $\mathcal{E}(\underline{t}, \overline{t}) = \{\tilde{a} | y^S(t, \tilde{a}) \geq U^S\ \forall t \in [\underline{t}, \overline{t}]\}$. Then, we obtain the following result:

Lemma 7

$$\gamma_-^S(\hat{t}_{\hat{N}-1}) \in \arg\max_{a\in\mathcal{E}(\hat{t}_{\hat{N}},\hat{t}_{\hat{N}-1})} V^R(a;\hat{t}_{\hat{N}},\hat{t}_{\hat{N}-1})$$

holds.

Proof:

It is evident when $\gamma_+(\hat{t}_{\hat{N}}) = \gamma_-(\hat{t}_{\hat{N}-1})$, since $\mathcal{E}(\hat{t}_{\hat{N}},\hat{t}_{\hat{N}-1}) = \{\gamma_-(\hat{t}_{\hat{N}-1})\}$. Consider the case where $\gamma_+(\hat{t}_{\hat{N}}) \neq \gamma_-(\hat{t}_{\hat{N}-1})$. On the basis of the construction of $\{\hat{t}_n\}$ and Lemma 6, $\mathcal{E}(\hat{t}_{\hat{N}},\hat{t}_{\hat{N}-1}) = [\gamma_-(\hat{t}_{\hat{N}-1}), \gamma_+(\hat{t}_{\hat{N}})]$. Then, it is sufficient to show that $\gamma_-(\hat{t}_{\hat{N}-1}) \geq \sigma^R(\hat{t}_{\hat{N}-1})$ since $V^R(a;\hat{t}_{\hat{N}},\hat{t}_{\hat{N}-1})$ is decreasing in a on $[\sigma^R(\hat{t}_{\hat{N}-1}), \infty)$.

On the basis of the presupposition,

$$\gamma_-(\hat{t}_{\hat{N}-1}) > \sigma(\hat{t}_{\hat{N}-1}) - \frac{\underline{\delta}}{2(N-1)}$$

holds. Meanwhile, on the basis of Assumption 1 and Lemma 4,

$$\sigma^S(\hat{t}_{\hat{N}-1}) - \sigma^R(\hat{t}_{\hat{N}-1}) \geq \sigma^S(\hat{t}_{\hat{N}}) - \sigma^R(\hat{t}_{\hat{N}}) - \overline{\delta}(\hat{t}_{\hat{N}-1} - \hat{t}_{\hat{N}}) > b - \frac{\overline{\delta}}{N-1}$$

holds. Then, we obtain

$$\begin{aligned}\gamma_-(\hat{t}_{\hat{N}-1}) &> \sigma^S(\hat{t}_{\hat{N}-1}) - \frac{\underline{\delta}}{2(N-1)}\\ &> \sigma^R(\hat{t}_{\hat{N}-1}) + b - \frac{\underline{\delta}}{2(N-1)} - \frac{\overline{\delta}}{N-1}\\ &\geq \sigma^R(\hat{t}_{\hat{N}-1}).\end{aligned}$$

This completes the proof. ∎

Let us return to the proof of Theorem 2. We define $\{\hat{a}_n\}_{n=1}^{\hat{N}}$ as follows:

$$\hat{a}_n = \gamma_-^S(\hat{t}_{n-1}).$$

Then, we construct a candidate for an equilibrium, $(\mu, p, \alpha, \epsilon)$, as follows:

$$\begin{aligned}&\{\tau_n\}_{n=1,\ldots,N} \text{ is a partition of } [0,1],\\ &\inf\tau_n = \hat{t}_n \text{ and } \sup\tau_n = \hat{t}_{n-1}, \quad n = 1,\ldots,N,\\ &\mu(t) = m_n, \quad \text{if } t\in\tau_n,\\ &p(m_n,t) = f_{\tau_n}(t),\\ &\alpha(m_n) = \hat{a}_n,\\ &\epsilon(t,a) = 0, \quad \text{iff } y^S(t,a) \geq U^S(t).\end{aligned}$$

From Lemmas 6 and 7, R has no incentive to deviate from α as long as R's equilibrium payoff is sufficiently larger than her exit payoff. Moreover, it is easily verified that S in $t \in \tau_n$ has no incentive to send message $m_{\tilde{n}}$ for $\tilde{n} \neq n$. Then, $(\mu, p, \alpha, \epsilon)$ constitutes an equilibrium. ∎

A.7 Proof of Corollary 3

It is easily verified that $\gamma_+(t) - \gamma_-(t) \to 0$ as $D^S \to 0$. Then, it is sufficient to show that Assumption 2 holds. We prove only (a) (we can similarly prove (b)). Suppose, to the contrary, that there exists $t > t'$ such that $\gamma_-(t) \leq \gamma_-(t')$. Then, we obtain

$$\begin{aligned}
D^S &= y^S(t', \sigma^S(t')) - U^S(t') \\
&= \int_{\gamma_-(t')}^{\sigma^S(t')} \frac{\partial y^S(t', a)}{\partial a} da \\
&< \int_{\gamma_-(t')}^{\sigma^S(t')} \frac{\partial y^S(t, a)}{\partial a} da \\
&\leq \int_{\gamma_-(t)}^{\sigma^S(t)} \frac{\partial y^S(t, a)}{\partial a} da \\
&= y^S(t, \sigma^S(t)) - U^S(t) \\
&= D^S.
\end{aligned}$$

Note that the fourth inequality holds since $\gamma_-(t) \leq \gamma_-(t')$, $\sigma^S(t) > \sigma^S(t')$, and $\frac{\partial y^S(t,a)}{\partial a} \geq 0$ for any $a \leq \sigma^S(t)$. This is a contradiction. ∎

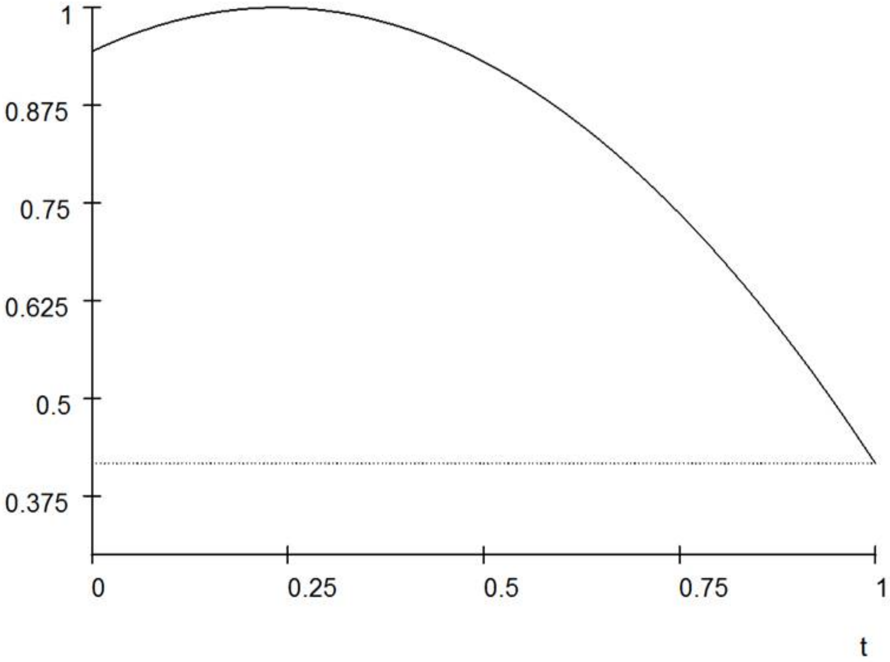

Figure 1: $U^S = \left(\frac{3}{2}+b\right)\left(\frac{1}{2}-b\right)$

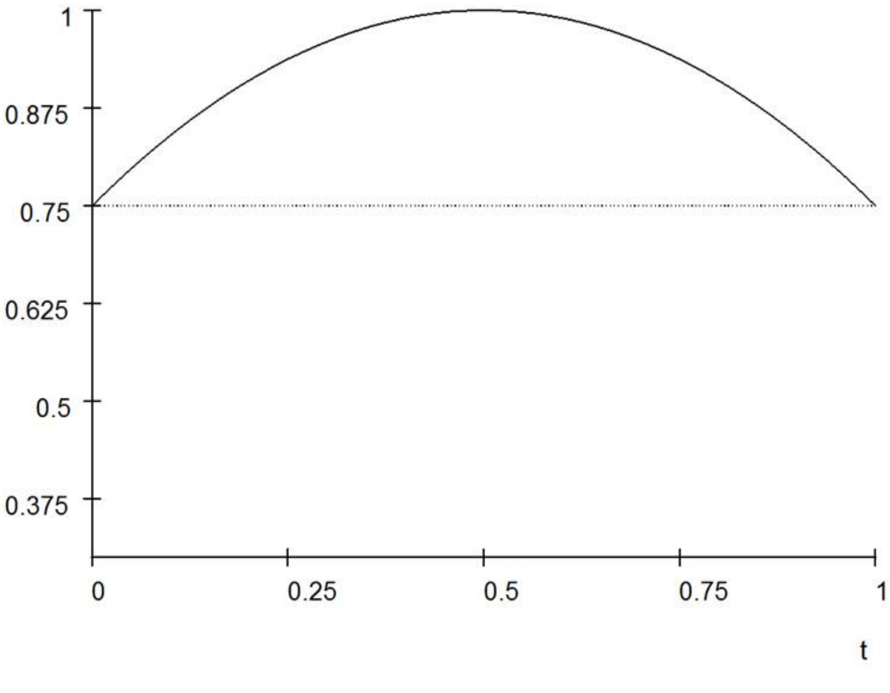

Figure 2: $U^S = \frac{3}{4}$

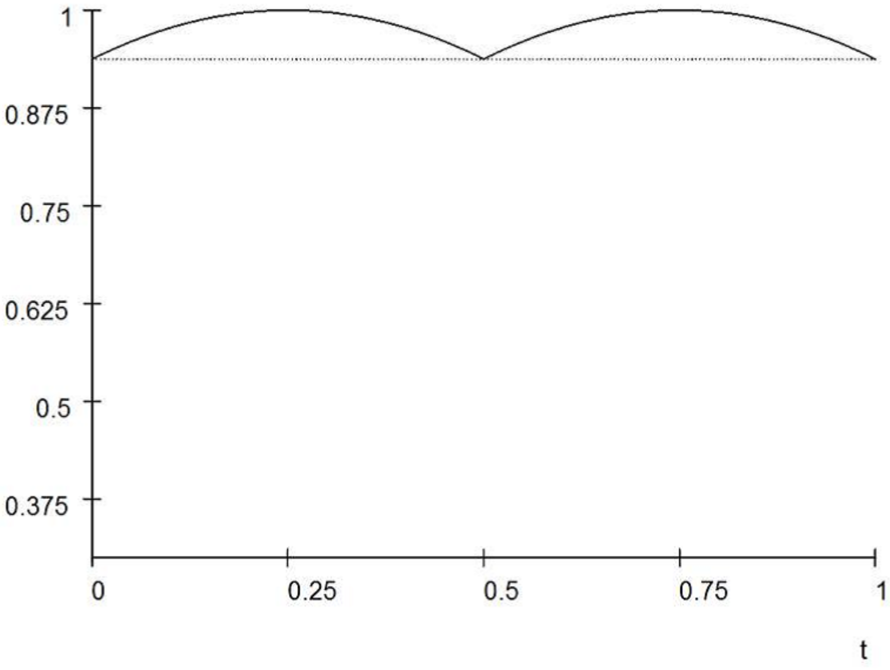

Figure 3: $U^S = \frac{15}{16}$

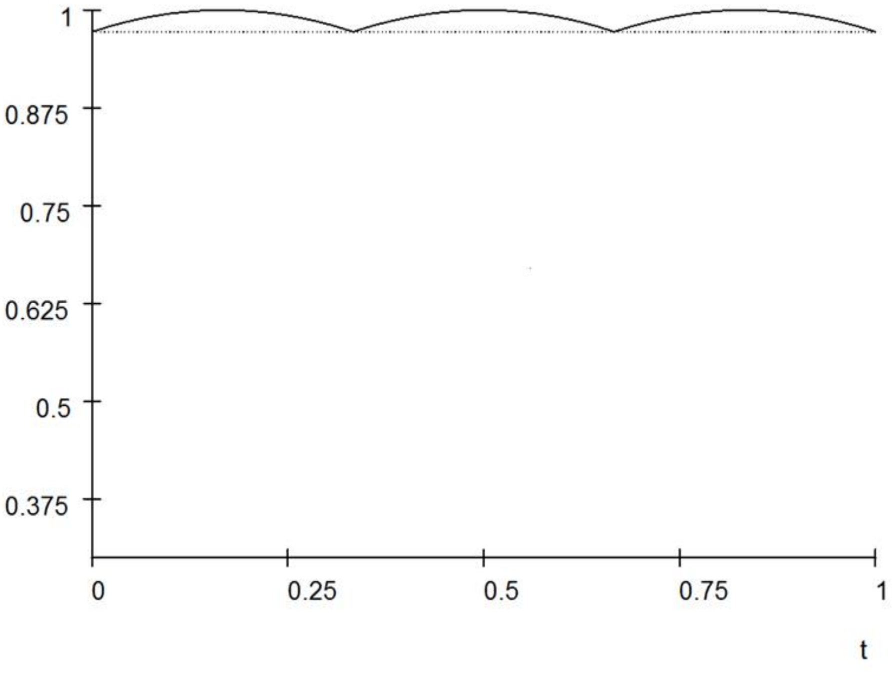

Figure 4: $U^S = \frac{35}{36}$

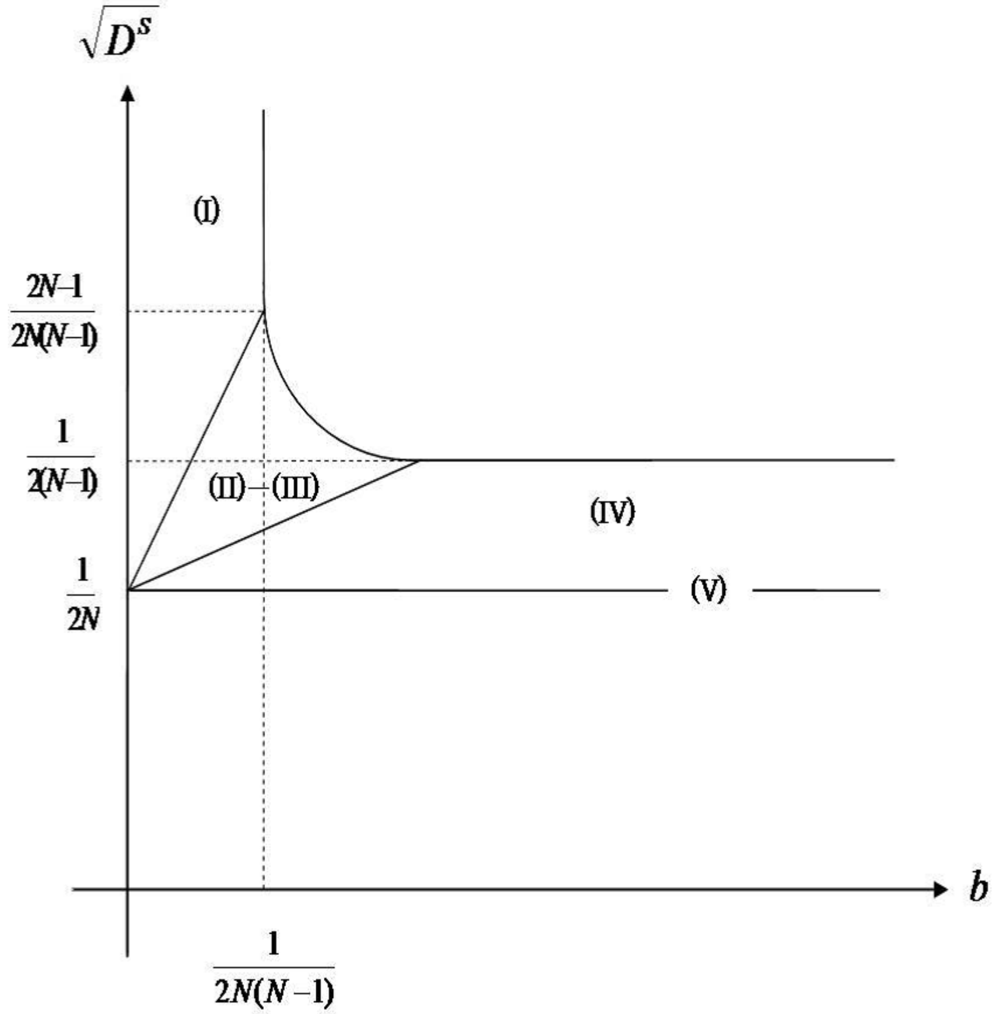

Figure 5: Equilibrium condition of NEE with N intervals ($N \geq 2$)