# Cheap Talk with the Exit Option: A Model of Exit and Voice

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# Modeling “Exit and Voice”

- Exit and voice framework (Hirschman 1970)
- Exit = dissolution of relationship
- Voice = cheap talk à la Crawford and Sobel (1982)

# Cheap Talk with an Exit Option

- Sender has an exit option after Receiver's action choice
- Exit payoffs are independent from Receiver's action
- The key variable is the difference between the maximum stay payoff (the stay payoff when the optimal action is chosen) and the exit payoff

# Main Results

- Consider the case where Sender's difference is positive but small while Receiver's difference is large
- Sender is likely to choose to exit, while Receiver has a strong incentive to prevent Sender from doing so
- Receiver is expected to choose an action desirable for Sender
- Sender has an incentive to send a more accurate information

# Main Results (cont'd)

- Existence of Sender's exit option enhances the informativeness of cheap talk, and increases Receiver's payoff as well as Sender's
- Perfect information transmission can be approximately attained as Sender's difference uniformly goes to 0
- New determinant of informativeness of cheap talk = credibility of exit

# Players and Timing

- The Sender and the Receiver
- Nature randomly chooses a state $\mathbf{t} \in \mathbf{T}$ according to a distribution function $\mathbf{F(t)}$
- $\mathbf{t}$ is observed only by S
- S costlessly sends a message $\mathbf{m} \in \mathbf{M}$ to R
- R chooses an action $\mathbf{a} \in \mathbf{A}$
- S chooses whether to stay or exit

# Payoffs

- $T = M = [0, 1]$
- $A = \mathbb{R}$
- $F(t)$ has a density with full support
- $i$'s stay payoff: $y^i(t, a)$
  - $\partial y^i / \partial a$: $C^1$
  - single-peaked in $a$
  - single-crossing ($\partial^2 y^i / \partial a \partial t > 0$)
- $i$'s exit payoff: $U^i(t)$
  - $U^S(t)$: $C^1$

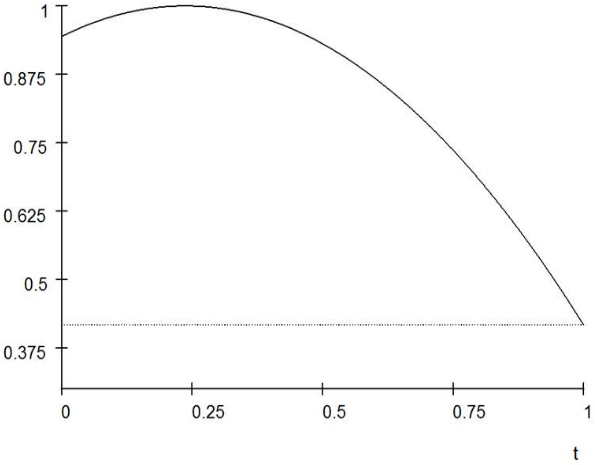
# Uniform-Quadratic Model with Constant Difference

- $\mathbf{F(t)}$ is uniform on $\mathbf{[0,1]}$
- $\mathbf{y^R(t,a) = Y^R - (t-a)^2}$
- $\mathbf{y^S(t,a) = Y^S - (t+b-a)^2}$
- $\mathbf{b}$: bias, degree of incongruence between S's and R's optimal actions
- We assume $\mathbf{b > 0}$
- $\mathbf{U^i}$: constant
- Difference between maximum stay payoff and exit payoff: $\mathbf{D^i = Y^i - U^i}$
- Assume $\mathbf{D^S > 0}$ and $\mathbf{D^R > 0}$

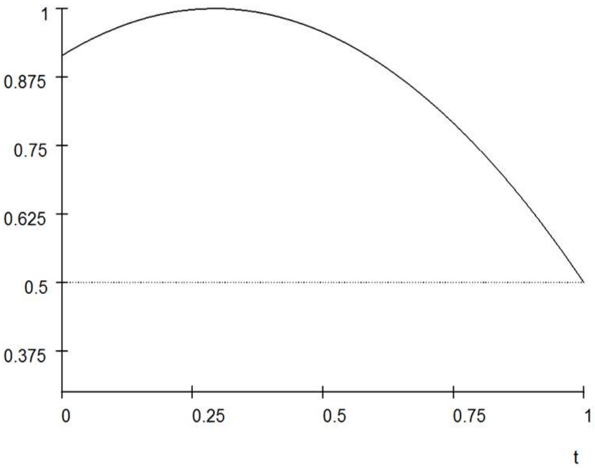
# Example

- $Y^S = 1$
- $b = \sqrt{10}/12$
- In the environment without exit, the unique equilibrium is babbling
- In the environment with exit, when $U^R$ is sufficiently small
- If $U^S \leq 0.417$, the unique equilibrium is the same as one without exit

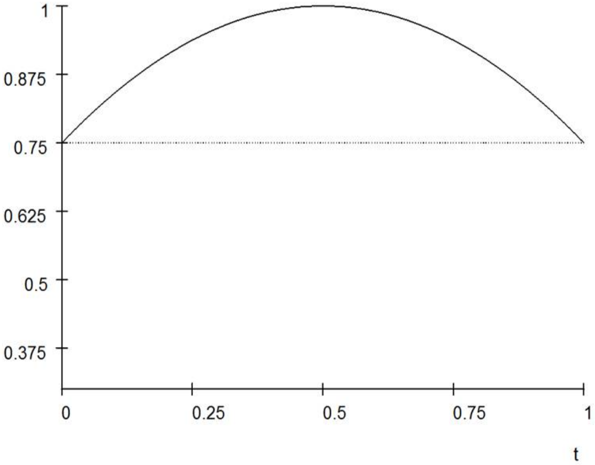
# $U^S = 0.417$



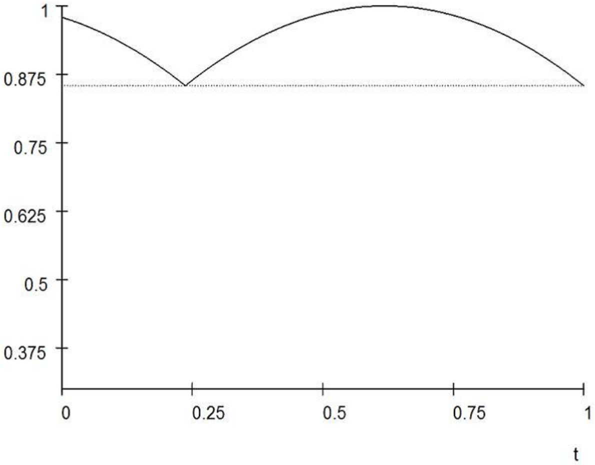
# $U^S = 0.5$



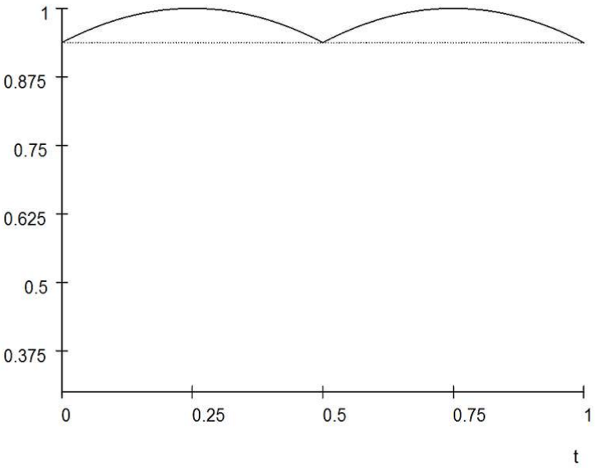
# $U^S = 0.75$



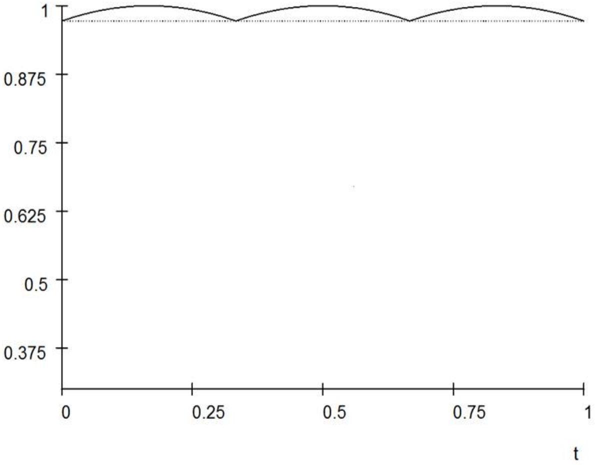
# $U^S = 0.854$



# $U^S = 0.937$



# $U^S = 0.972$



# No-Exit Equilibrium

- We characterize the class of No-Exit Equilibria (NEE)
- Lemma 1: Any NEE is characterized by finite intervals partitioning the state space

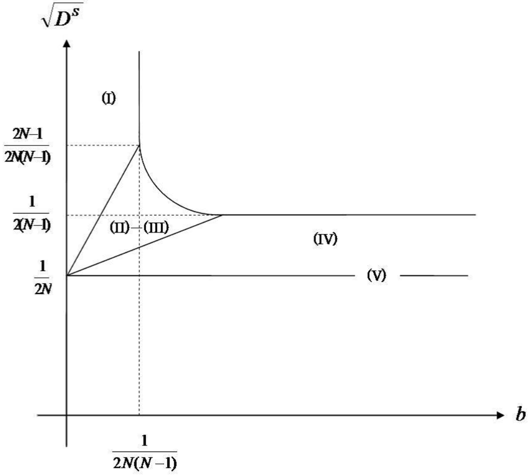
# Lemma2: 3 Types of Intervals

- Any interval $(\underline{\mathbf{t}}, \bar{\mathbf{t}})$ with $\hat{\mathbf{a}}$ is classified into
  1. $\mathcal{F}$ully accomodating interval:
     $\mathbf{y^S(\underline{t}, \hat{a}) = y^S(\bar{t}, \hat{a}) = U^S}$
  2. $\mathcal{A}$ccomodating interval:
     $\mathbf{y^S(\underline{t}, \hat{a}) > U^S}$, $\mathbf{y^S(\bar{t}, \hat{a}) = U^S}$
  3. $\mathcal{N}$on-accomodating interval:
     $\mathbf{y^S(\underline{t}, \hat{a}) > U^S}$, $\mathbf{y^S(\bar{t}, \hat{a}) > U^S}$
- R's incentive is satisfied if $\sqrt{\mathbf{D^R}} \geq \sqrt{\mathbf{D^S}} + \mathbf{b}$

# Lemma 3: Possible Configurations of Intervals

- Possible configurations of intervals are either one of
  1. $\mathcal{N}, \ldots, \mathcal{N}$
  2. $\mathcal{N}, \ldots, \mathcal{N}, \mathcal{A}$
  3. $\mathcal{N}, \ldots, \mathcal{N}, \mathcal{A}, \mathcal{F}, \ldots, \mathcal{F}$
  4. $\mathcal{A}, \mathcal{F}, \ldots, \mathcal{F}$
  5. $\mathcal{F}, \ldots, \mathcal{F}$
- S's incentive condition is obtained (omitted)

# Theorem 1: Condition for NEE with **N** Intervals



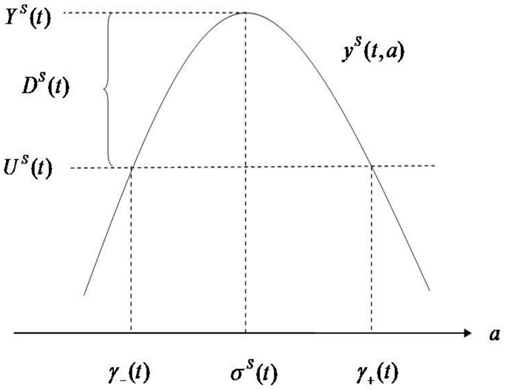
# 2 Determinants of Informativeness of Cheap Talk

- Congruence of preferences (small $\mathbf{b}$)
- Credibility of exit (small $\mathbf{D^S}$)
  $\mathbf{D^S} \downarrow$
  $\Rightarrow$ the credibility of exit $\uparrow$
  $\Rightarrow \mathbf{N} \uparrow$

# Corollaries

- Suppose $\sqrt{\mathbf{D^R}} > \mathbf{b}$
- Corollary 1: There exists a sequence of equilibria in which the equilibrium actions induced by $\mathbf{t}$ converge to $\mathbf{t+b}$ as $\mathbf{D^S} \rightarrow \mathbf{0}$
- Collorary 2: Assume $\mathbf{b} < \mathbf{1/2\sqrt{3}}$. There exists an NEE that gives larger ex ante payoffs to both players than the maximal payoffs obtained in the environment without exit if $\mathbf{D^S}$ is sufficiently small

# General Model: Notations



# Assumptions

- Assumption 1: Either one of the following condition hold:

  (a) $\boldsymbol{\sigma}^{\mathsf{S}}(\mathbf{0}) > \boldsymbol{\sigma}^{\mathsf{R}}(\mathbf{0})$
  (b) $\boldsymbol{\sigma}^{\mathsf{R}}(\mathbf{1}) > \boldsymbol{\sigma}^{\mathsf{S}}(\mathbf{1})$

- Denote $\mathbf{b} = \boldsymbol{\sigma}^{\mathsf{S}}(\mathbf{0}) - \boldsymbol{\sigma}^{\mathsf{R}}(\mathbf{0})$ or $\mathbf{b} = \boldsymbol{\sigma}^{\mathsf{R}}(\mathbf{1}) - \boldsymbol{\sigma}^{\mathsf{S}}(\mathbf{1})$
- Assumption 2: Under Assumtion 1,
  - 1(a) is required $\Rightarrow \gamma_{-}(\mathbf{t})$: strictly increasing
  - 1(b) is required $\Rightarrow \gamma_{+}(\mathbf{t})$: strictly increasing

# Theorem 2: Sufficient Condition for NEE with $\mathbf{N}$ or More Intervals

- Under Assumptions 1 and 2, suppose $\mathbf{D^R(t)}$ is sufficiently large and $\mathbf{D^S(t) > 0}$ for any $\mathbf{t}$. Then, for any $\mathbf{N > \underline{N}}$, there exists an NEE with $\mathbf{N}$ or more intervals if $\gamma_+(\mathbf{t}) - \gamma_-(\mathbf{t}) \leq \overline{\gamma}$ for any $\mathbf{t}$ where
  - $\mathbf{\underline{N}} = \frac{\underline{\delta}+2\overline{\delta}}{\mathbf{2b}}$
  - $\overline{\gamma} = \frac{\underline{\delta}}{\mathbf{2(N-1)}}$
  - $\underline{\delta} = \inf_{\mathbf{t>t'}} \frac{\sigma^{\mathbf{S}}(\mathbf{t})-\sigma^{\mathbf{S}}(\mathbf{t'})}{\mathbf{t-t'}}$ $(> \mathbf{0})$
  - $\overline{\delta} = \sup_{\mathbf{t>t'}} \frac{\sigma^{\mathbf{R}}(\mathbf{t})-\sigma^{\mathbf{R}}(\mathbf{t'})}{\mathbf{t-t'}}$ $(< \infty)$
- The length of each interval can be made shorter than $\frac{\mathbf{1}}{\mathbf{N-1}}$
- This theorem doesn't require any assumption on distribution function

# Corollary 3: Limit Result in General Model

- Under Assumption 1, suppose $\mathbf{D^R(t)}$ is sufficiently large for any $\mathbf{t}$. If $\mathbf{D^S(t)}$ is constant, then there exists a sequence of equilibria in which the equilibrium actions induced by $\mathbf{t}$ converge to $\mathbf{t+b}$ as $\mathbf{D^S \to 0}$

# Example 1: Preference Reversal

- $\mathbf{U^i}$: constant
- $\mathbf{y^R = Y^R - (ct - b - a)^2}$
- $\mathbf{y^S = Y^S - (t - a)^2}$
- $\mathbf{b, c > 0}$
- $\mathbf{c > 1 + b}$ $\Rightarrow$ preference reversal: $\sigma^R(0) < \sigma^S(0)$ , $\sigma^R(1) > \sigma^S(1)$
- Corollary 3 applies

# Example 2: Quadratic Model with Variable Difference

- $y^S = Y^S(t) - (t + b - a)^2$
- $y^R = Y^R(t) - (t - a)^2$
- $b > 0$
- $\frac{1}{4(N-1)} \geq \sqrt{D^S(t)} > \frac{1}{2}\frac{dD^S}{dt}$
  $\Rightarrow \exists$ NEE with **N** or more intervals

# Conclusion

- I present a cheap talk model with an exit option
- Existence of sender's exit option can increase the informativeness of cheap talk and both players' payoffs
- Perfect information transmission via cheap talk can be approximately attained in the limit
- New determinant of informativeness of cheap talk = credibility of exit

# Related Literature

- Crawford and Sobel (1982): our basic model
- Matthews (1989): private information about S's preference
- Banerjee and Somanathan (2001): deal with a collective aspect of voice formation, but not with exit
- Gehlbach (2006): no asymmetric information

# Related Literature (cont'd)

- Mechanism design
  Holmstrom (1984), Melumad and Shibano (1991), etc.
  Our paper: a simple contract allocating the joint surplus makes an efficiency outcome possible even if the sender cannot commit to message-dependent mechanisms
- Delegation without mechanism
  Dessein (2002) and Marino (2007)
  Our paper: a similar outcome arises while we don't allow delegation

# Strategies and Equilibrium

- Perfect Bayesian equilibrium with pure strategies: $(\mu, \mathbf{P}, \alpha, \epsilon)$
- $\mu : \mathbf{T} \longrightarrow \mathbf{M}$: S's message strategy
- $\mathbf{P} : \mathbf{M} \times \mathbf{T} \longrightarrow [0, 1]$: R's posterior belief distribution function over $\mathbf{T}$
- $\alpha : \mathbf{M} \longrightarrow \mathbf{A}$: R's action strategy
- $\epsilon : \mathbf{T} \times \mathbf{A} \longrightarrow \{0, 1\}$: S's exit strategy

# Case of $\mathbf{D^S = 0}$ ($\mathbf{Y^S = U^S}$)

- Perfect information transmission is possible with the equilibrium in which
  - $\boldsymbol{\mu}(\mathbf{t}) = \mathbf{t}$
  - $\boldsymbol{\alpha}(\mathbf{m}) = \mathbf{m} + \mathbf{b}$
  - $\boldsymbol{\epsilon}(\mathbf{t}, \boldsymbol{\alpha} \circ \boldsymbol{\mu}(\mathbf{t})) = \mathbf{0}$
- R's payoff $= \mathbf{Y^R} - \mathbf{b^2}$
- Equilibrium condition: $\mathbf{Y^R} - \mathbf{b^2} \geq \mathbf{U^R} \Leftrightarrow \mathbf{D^R} \geq \mathbf{b^2}$
- This payoff is larger than the maximal payoff obtained in the environment without exit

# Allocating $\mathbf{Y^S}$ and $\mathbf{Y^R}$

- Suppose gross joint surplus $\mathbf{Y}$ can be allocated into S's share $\mathbf{Y^S}$ and R's share $\mathbf{Y^R}$
- Consider the case where $\mathbf{Y} > \mathbf{U^S} + \mathbf{U^R} + \mathbf{b}^2$
- A simple contract with $\mathbf{Y^S} = \mathbf{U^S} + \varepsilon$ for small $\varepsilon$ makes an almost perfect information transmission possible

# Future Research

- Euilibrium characterization, including equilibria where exit is chosen on the equilibrium path
- Possibility of mechanism design / contract
- More general class of games with an exit option
- Multiple senders