Learnability of Heterogeneous Misspecification Equilibrium

Supplementary Material

(not intended for publication)

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Abstract

This appendix is prepared for the manuscript "Learnability of Heterogeneous Misspecification Equilibrium" to clarify that if all exogenous variables are equivalent, an equilibrium under HM learning is equivalent to an equilibrium under CS learning.[1]

Equilibrium under CS learning

We make an assumption about information sets held by agents under CS learning:

Assumption 1 *The information sets held by agents in time t include the lagged and current information of y_t and w_t, $\{y_s, w_s\}_{s=1}^{t}$.*

This information set is the same as information sets held by agents under rational expectations in that agents' forecast y_{t+1}^e is formulated with the information of not only past, but also current endogenous variable y_t. The difference is solely that agents in this paper have no information about the structure of the economy.

With the information set $\{y_s, w_s\}_{s=1}^{t}$, agents under adaptive learning formulate their forecast y_{t+1}^e like econometricians. First, agents form a CS PLM that has the same form as the MSV solution (3),

$$y_t = a + cw_t + \varepsilon_t, \tag{A1}$$

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[1] Equation numbers that are referred in this appendix point to the equations numbered in the manuscript, if the equations that have those numbers are not seen in this appendix.

where ε_t is a $m \times 1$ vector of error terms that are believed to be white noise. Next, agents estimate the parameter matrix $\phi' = (a, c)$ using a recursive least-squares (RLS) method. Finally, using the CS PLM (A1) and the estimated ϕ, agents formulate the forecast y^e_{t+1} as

$$y^e_{t+1} = a + c\Phi_n w_t. \tag{A2}$$

Meanwhile, an actual law of motion (ALM) of the economy depends upon y^e_{t+1} in Eq. (A2). Then, substituting Eq. (A2) into Eqs. (1)-(2), the ALM is determined by

$$y_t = (A + Ba) + (Bc\Phi_n + C)\, w_t. \tag{A3}$$

The stability of this economy depends upon whether the estimate ϕ converges to the coefficient matrices $(A + Ba, Bc\Phi_n + C)$ in the ALM. In real-time learning, ϕ is updated recursively using newly available data every time. According to the *E-stability principle* found by Evans and Honkapohja (2001, chapter 2), the global convergence of ϕ in real-time learning through least-squares techniques is governed by the ordinary differential equation (ODE) derived by the mapping from the PLM (A1) to the ALM (A3),

$$\frac{d}{d\tau}(a, c) = T(a, c) - (a, c), \tag{A4}$$

where $T(a, c) \equiv (T_a(a), T_c(c)) = (A + Ba, Bc\Phi_n + C)$, and τ denotes notional time. Thus, if and only if the ODE (A4) is globally asymptotically stable, the economy is stable with ϕ converging to the fixed point:

$$\bar{a} = (I_m - B)^{-1} A, \tag{A5}$$

$$\bar{c} = B\bar{c}\Phi_n + C, \tag{A6}$$

which is an equilibrium to be attainable under CS learning.

We immediately find that when all exogenous variables are equivalent, the HME (13)-(14) is reduced to Eqs. (A5)-(A6). If $\sigma_{ii} = \sigma_{jj}$ and $\rho_{ij} = 1$ for all i, j, $\left(\frac{1}{n}\Psi_n\right) w_t = w_t$, and Eqs. (14) and (A6) provide the same $\bar{c}w_t$. Thus, both equilibria provide the same solution $(\bar{a}, \bar{c})$.

The clarification is complete.

References

Evans, G. W. and S. Honkapohja (2001): *Learning and Expectations in Macroeconomics*, Princeton University Press.